# MATHEMATICS
RESOURCE PACK
GRADE 12 TERM 1

# SEQUENCES AND SERIES

## RESOURCE 1

### LESSON 2

**1. ASSIGNMENT: SEQUENCES AND SERIES** **TOTAL: 60**

**INSTRUCTIONS**

1. Answer all the questions.
2. Clearly show all calculations you have used in determining your answers.
3. Round answers off to TWO decimal places, unless stated otherwise.
4. Number your answers correctly according to the numbering system used in this question paper.
5. Write neatly and legibly.

**QUESTION 1**

Lucy is arranging 1-cent and 5-cent coins in rows. The pattern of the coins in each row is shown below.


1.1 Calculate the total number of coins in the $40^{th}$ row. (3)

1.2 Calculate the total value of the coins in the $40^{th}$ row. (4)

1.3 Which row has coins with a total value of 337 cents? (6)

1.4 Show that the total value of the coins in the first 40 rows is 4 800 cents. (6)

**[19]**

**QUESTION 2**

The sum of the first n terms of a sequence is given by: $S_n = n(23 - 3n)$

| | | |
|---|---|---|
| 2.1 | Write down the first THREE terms of the sequence. | (5) |
| 2.2 | Calculate the $15^{th}$ term of the sequence. | (3) |
| | | **[8]** |

**QUESTION 3**

The sum of the second and third terms of a geometric sequence is 280, and the sum of the fifth and the sixth terms is 4 375. Determine:

| | | |
|---|---|---|
| 3.1 | The common ratio AND the first term. | (6) |
| 3.2 | The sum of the first 10 terms. | (2) |
| | | **[8]** |

**QUESTION 4**

Determine the value of *k* if:

$$\sum_{t=1}^{\infty} 4.k^{t-1} = 5$$ **[6]**

**QUESTION 5**

Given the series: $2(5)^5 + 2(5)^4 + 2(5)^3 + ...$

Show that this series converges. [2]

**QUESTION 6**

If 2; $x$; 18; ... are the first three terms of a geometric sequence, determine the value(s) of $x$. **[4]**

**QUESTION 7**

Given: $T_n = 3^{n+1}$. Which term is the first to exceed 20 000? **[4]**

**QUESTION 8**

The sequence 3; 9; 17; 27; ... is a quadratic sequence.

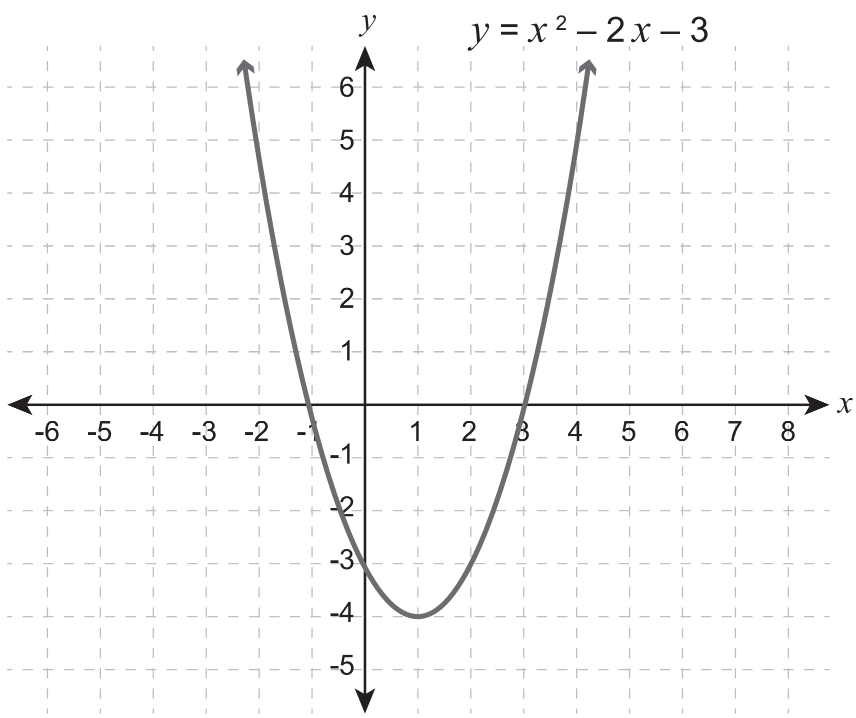
| | | |
|---|---|---|
| 8.1 | Write down the next term of the sequence. | (1) |
| 8.2 | Determine an expression for the $n^{th}$ term of the sequence. | (4) |
| 8.3 | What is the value of the first term of the sequence that is greater than 269? | (4) |
| | | **[9]** |

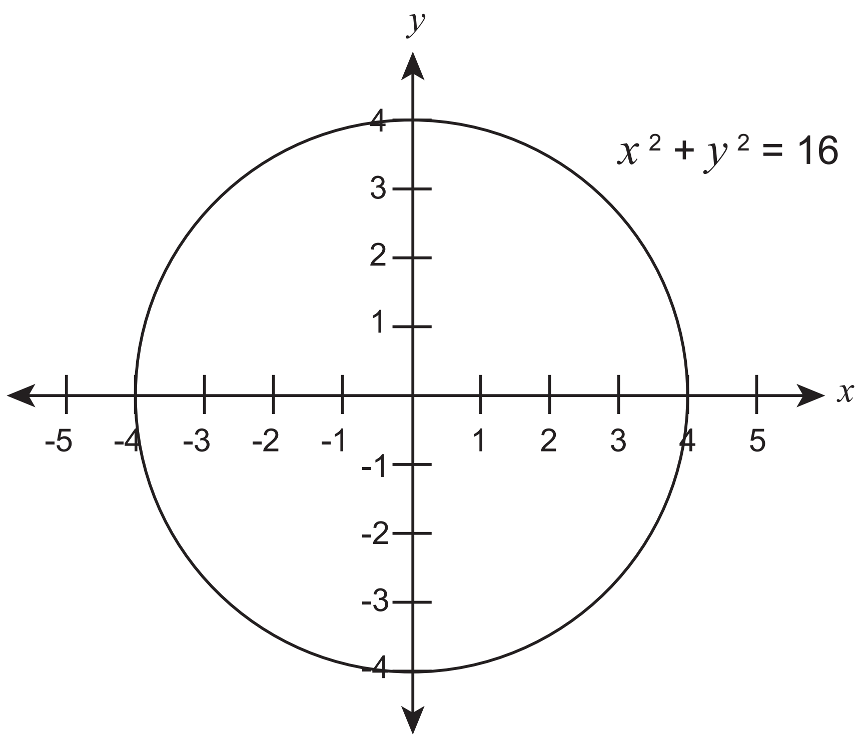
**TOTAL:60**

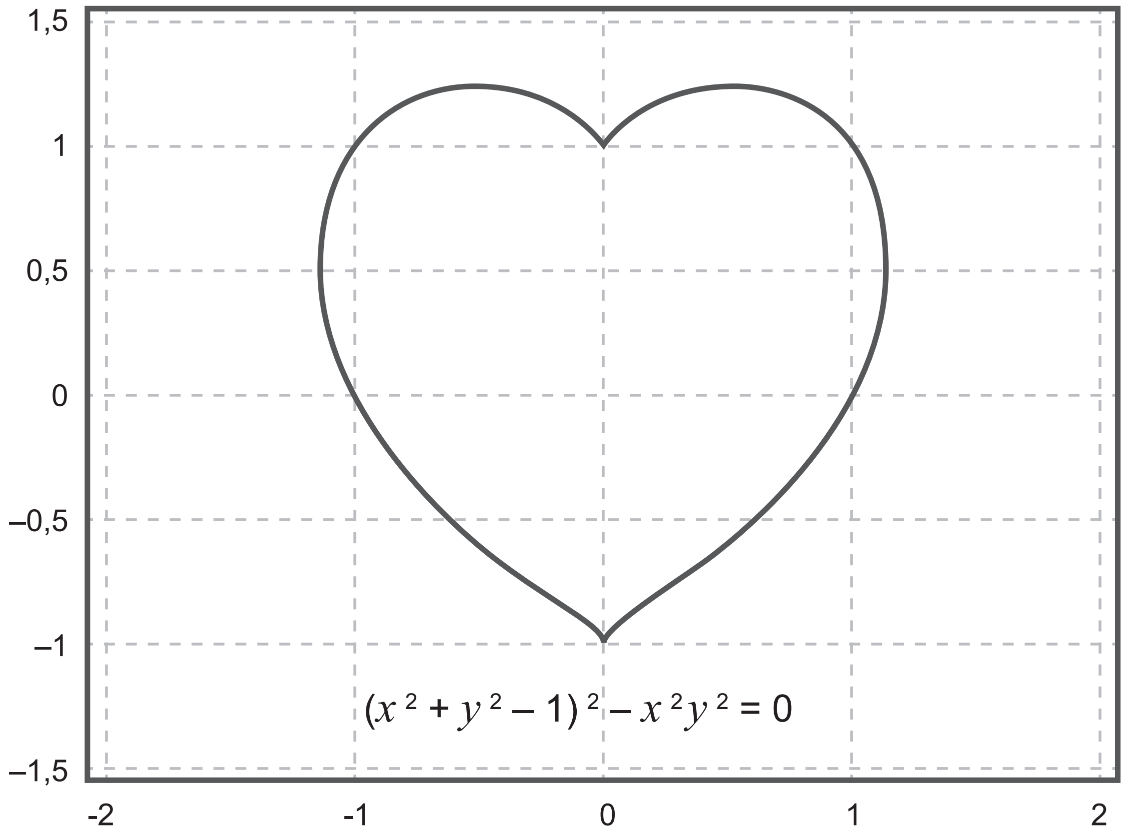
# FUNCTIONS

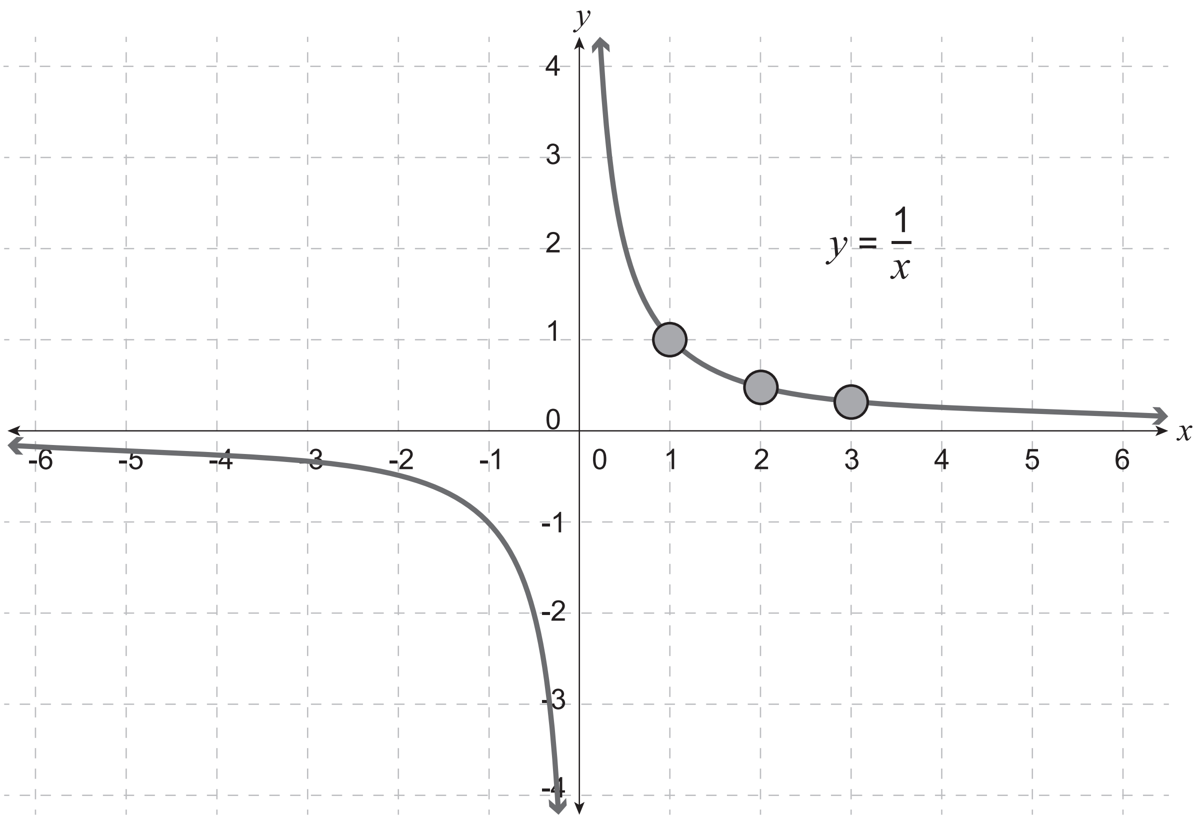
## RESOURCE 2

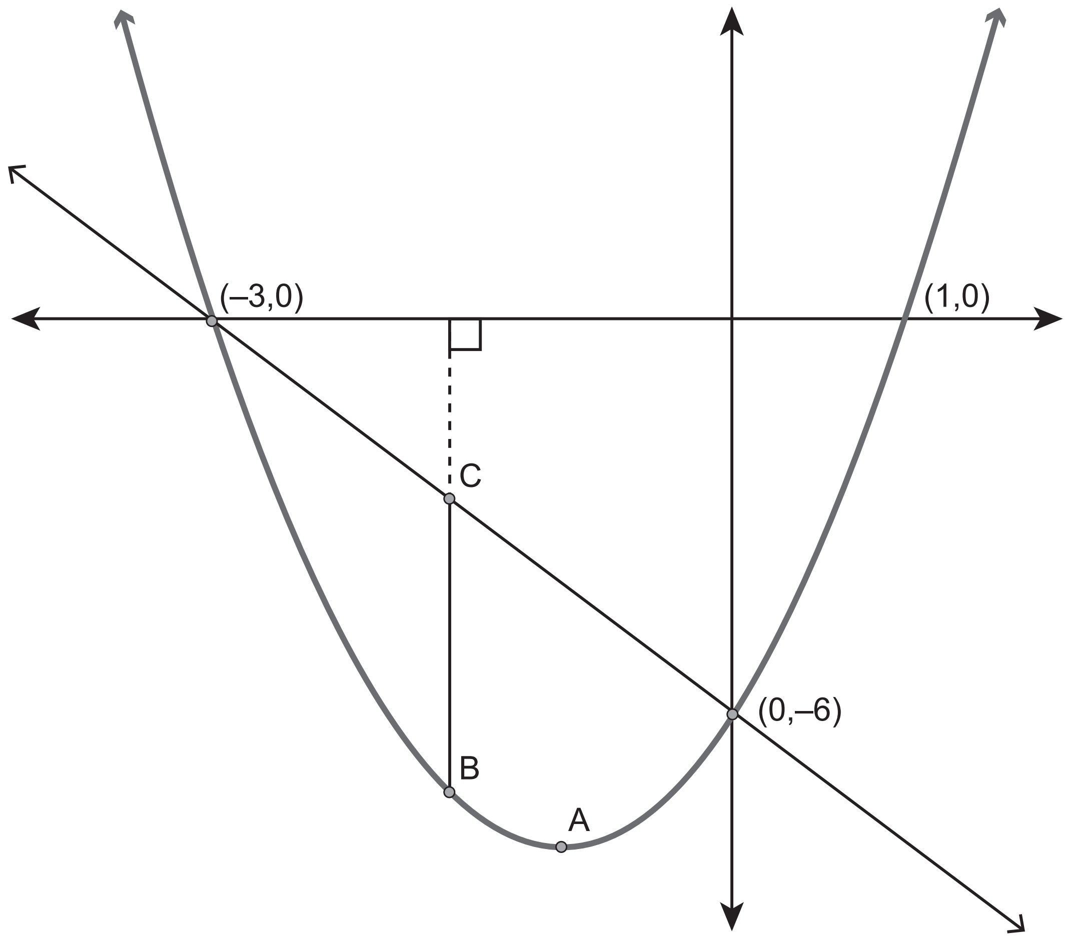
### LESSON 1

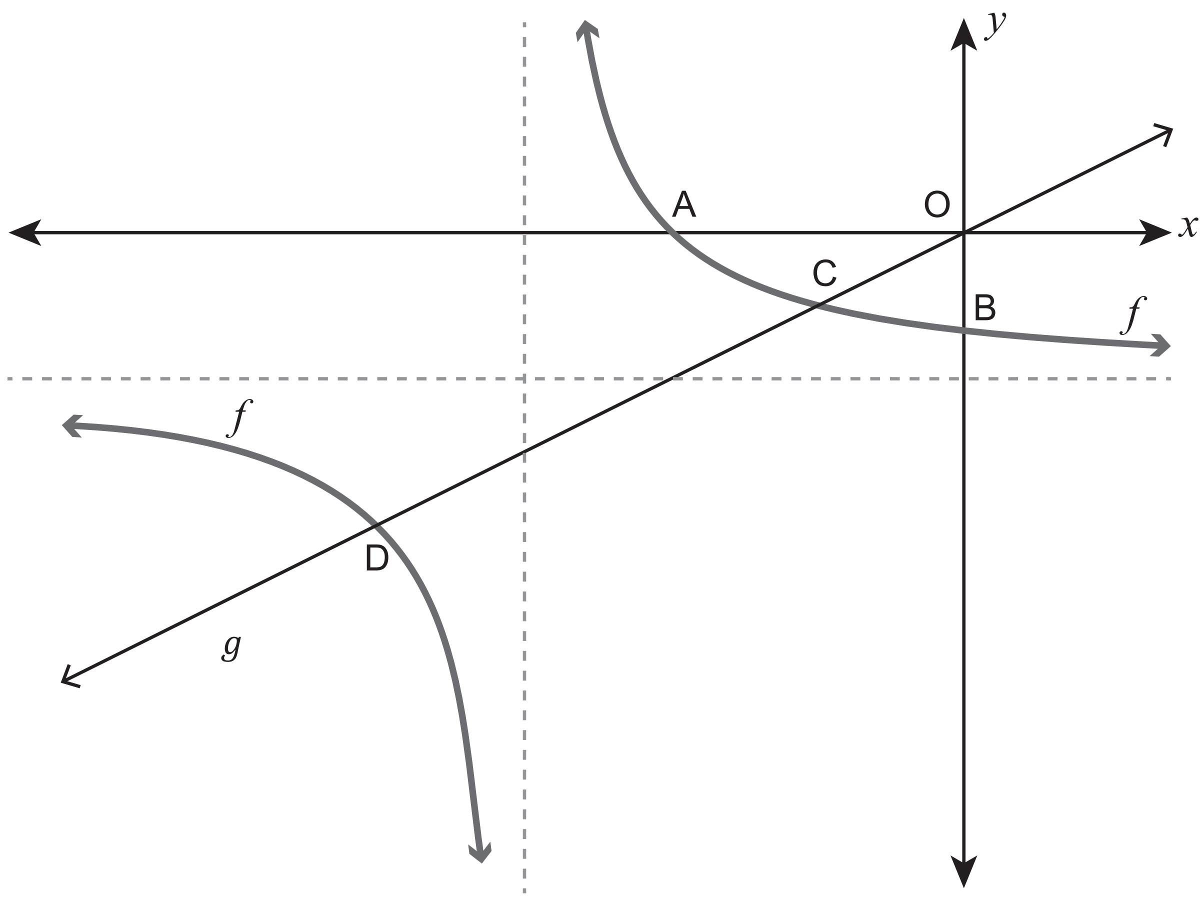


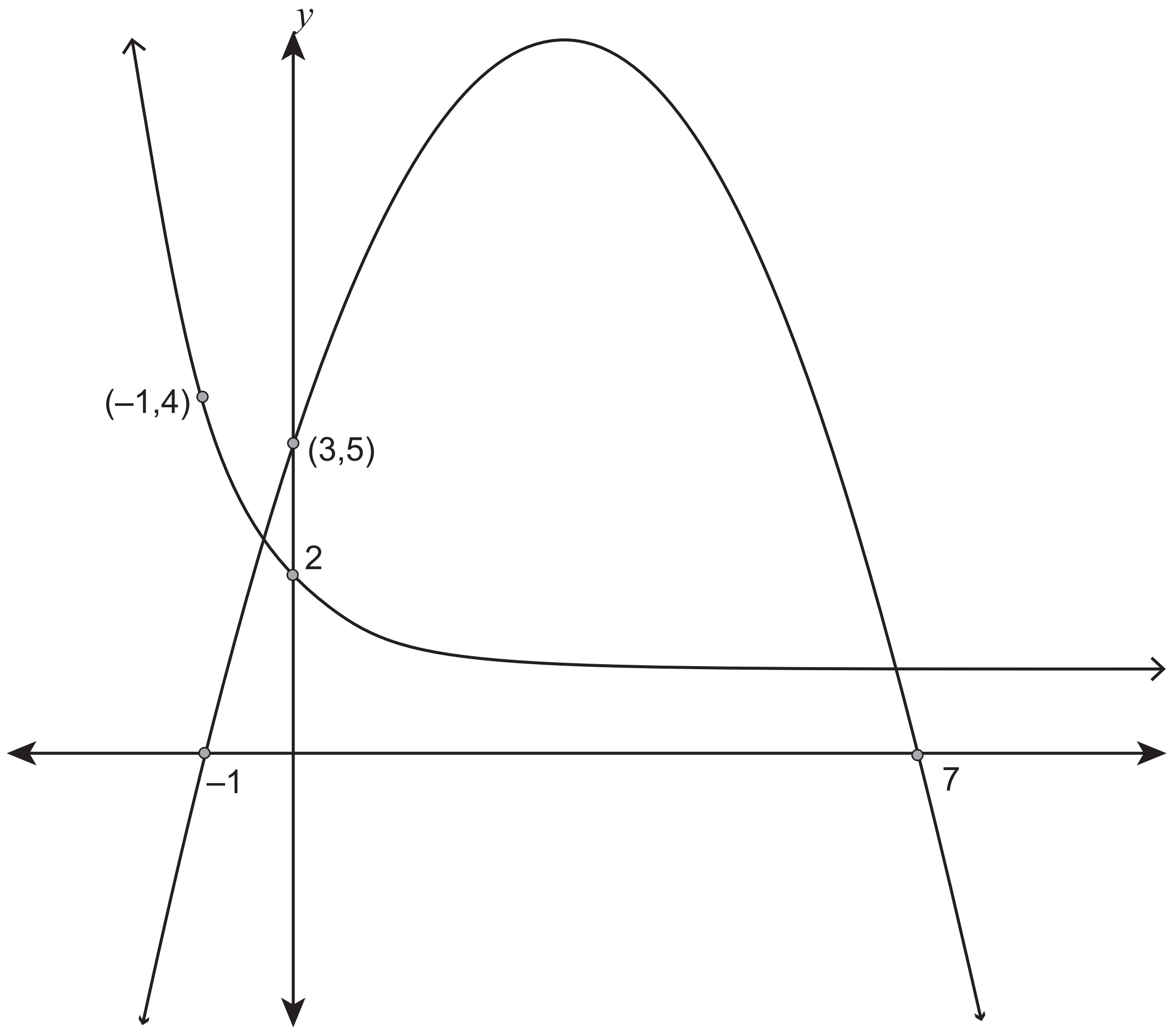




1,5
1
0,5
0
–0,5
–1
–1,5
-2
-1
0
1
2
$(x^2 + y^2 - 1)^2 - x^2y^2 = 0$


y
x
$y = \frac{1}{x}$
4
3
2
1
0
-1
-2
-3
-4
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6


(–3,0)
(1,0)
C
(0,–6)
B
A


y
x
A
O
C
B
f
f
D
g


y
(–1,4)
(3,5)
2
–1
7

# RESOURCE 3

## LESSON 2

**2. INVESTIGATION 1: FUNCTIONS AND INVERSES**

**TOTAL: 50**

**INSTRUCTIONS**

1. Answer all the questions.
2. Clearly show all calculations you have used in determining your answers.
3. Round answers off to TWO decimal places, unless stated otherwise.
4. Number your answers correctly according to the numbering system used in this question paper.
5. Write neatly and legibly.

**PART 1: WHICH RELATIONS CONSTITUTE FUNCTIONS?**

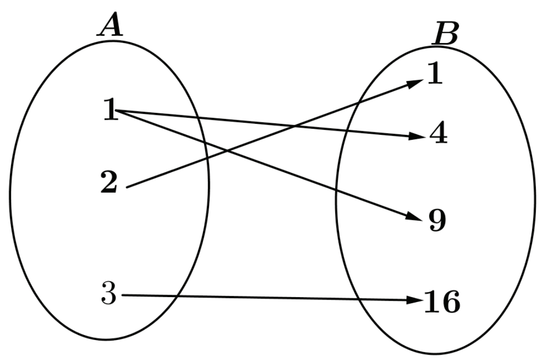
One-to-one relation: A relation is one-to-one if for every input value there is only one output value.

Many-to-one relation: A relation is many-to-one if for more than one input value there is one output value.

One-to-many relation: A relation is one-to-many if for one input value there is more than one output value.

1.1 Determine the type of relation in each case and give a reason.

1.1.1

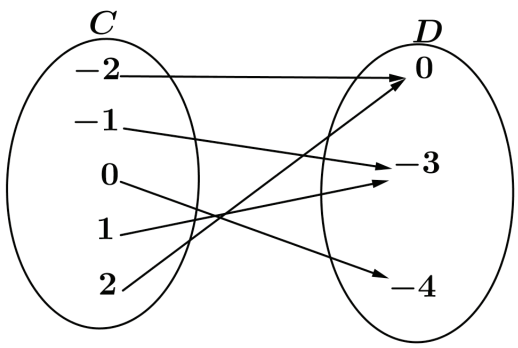



(1)

1.1.2 {(1 ; 3), (2 ; 5), (6 ; 13), (7 ; 15)}

(1)

1.1.3




______________________________________________________________ (1)

______________________________________________________________

A function is a set of ordered number pairs where no two ordered pairs have the same $x$-coordinate, or put differently: a function is a set of ordered pairs where, for every value of $x$ there is one and only one value for $y$. However, for the same value of $y$ there may be different values for $x$.

1.2 Which of the relations (in QUESTIONS 1.1.1 to 1.1.3) are functions? Why?

(a)______________________________________________________________ (2)

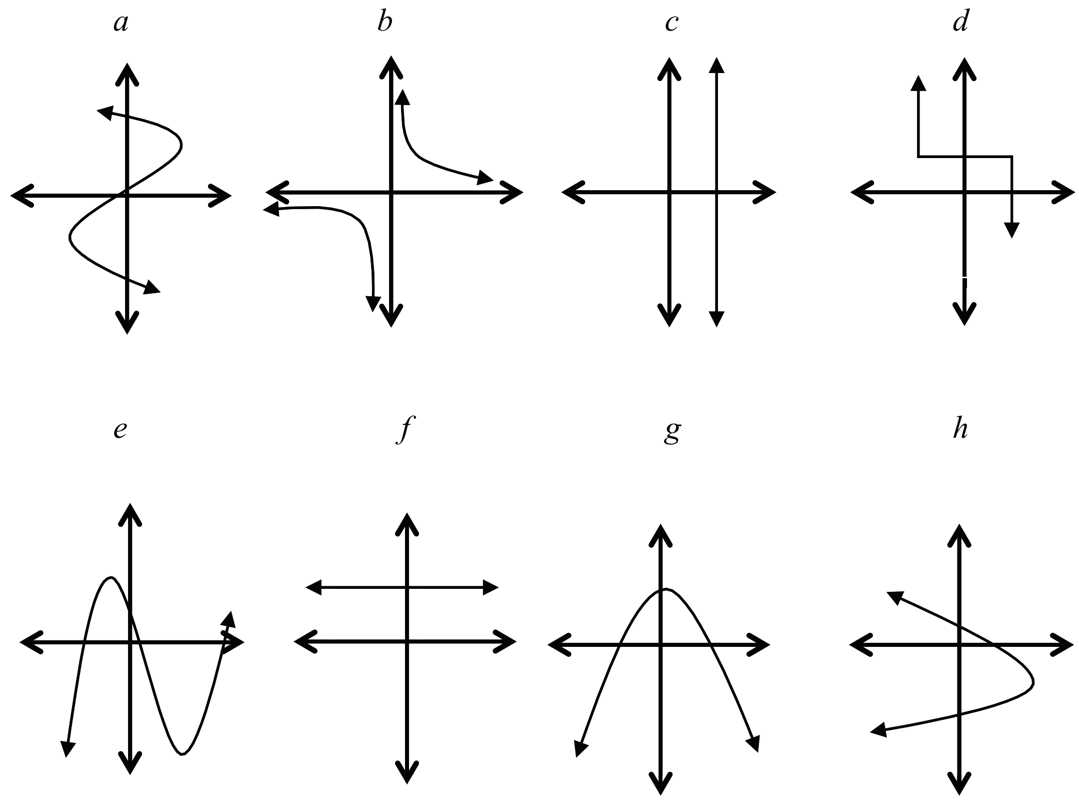
(b)______________________________________________________________ (1)

(c)______________________________________________________________ (1)

<u>The vertical-line test is used to determine whether or not a given graph is a function.</u>

To determine whether a graph is a function, do the vertical-line test. If any vertical line intersects the graph of $f$ only once, then $f$ is a function; and if any vertical line intersects the graph of $f$ more than once, then $f$ is not a function.

1.3 Determine whether or not the following graphs are functions. Give a reason for your answer.




(a)________________________________________________ (1)

(b)________________________________________________ (1)

(c)________________________________________________ (1)

(d)________________________________________________ (1)

(e)________________________________________________ (1)

(f)________________________________________________ (1)

(g)________________________________________________ (1)

(h)________________________________________________ (1)

## PART 2: THE INVERSE OF AN EXPONENTIAL FUNCTION

2.1 Consider the equation $g(x) = 2^x$. Now complete the following table:

| **x** | – 3 | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| **y** | | | | | | |

(1)

2.2 Sketch the graph of $g$.

2.3 Sketch the graph of $f(x) = x$ as a dotted line on the same set of axes as $g$. (1)

2.4 Complete the table below for $h$, if $h$ is $g$ when the $x$ and $y$ values are interchanged.

| **x** | | | | | | |
|---|---|---|---|---|---|---|
| **y** | | | | | | |

Draw $h$ on the same set of axes as $g$. (4)

2.5 Hence, write down the $x$-intercept of each of the following graphs below.

$y = 2^x$ $x = 2^y$

2.5.1______________________________________________________

2.5.2______________________________________________________ (2)

2.6 Write down the domain and range of:

2.6.1 $y = 2^x$

Domain: ____________________

Range: ____________________ (2)

2.6.2 $x = 2^y$

Domain:____________________

Range :____________________ (2)

2.6.3 What is the relationship between the domain and the range of the two graphs in 2.6.1 and 2.6.2

______________________________________________________

______________________________________________________ (1)

2.6.4 Are both graphs functions? Give a reason for your answer .

______________________________________________________ (2)

2.6.5 Write the equation of $x = 2^y$ in the form $y =$

______________________________________________________ (1)

2.6.6 Do you notice any line of symmetry in your sketch? What is the equation of this line?

______________________________________________________ (1)

2.6.7 In mathematics we call $h$ the inverse of $g$. Make a conjecture about the graph and its inverse.

______________________________________________________ (3)

**PART 3: WHEN IS THE INVERSE OF A QUADRATIC FUNCTION ALSO A FUNCTION?**

3.1 Given: $f(x) = 2x^2$, for $x \in \mathbb{R}$

(1)

3.1.1 Write down the equation of the inverse of $f$.

__________________________________________________

3.1.2 Write down the turning points of both $f$ and its inverse.

__________________________________________________ (2)

3.1.3 Sketch the graphs of $f$ and its inverse on the same set of axes.

(2)

3.1.4 Decide whether or not the inverse of $f$ is a function, and give a reason for your answer. (2)

__________________________________________________

3.1.5 Explain how you would restrict the domain of $f$ such that its inverse is a function.

__________________________________________________ (2)

3.1.6 Hence, write down the corresponding range of the inverse of $f$ if:

(a) $x \leq 0$ ____________________________________________ (1)

(b) $x \geq 0$ ____________________________________________ (1)

3.1.7 On separate sets of axes, sketch the graphs of the inverse of $f$ with restricted domains as in QUESTION 3.1.6. Indicate the domain and range of each.

(2)

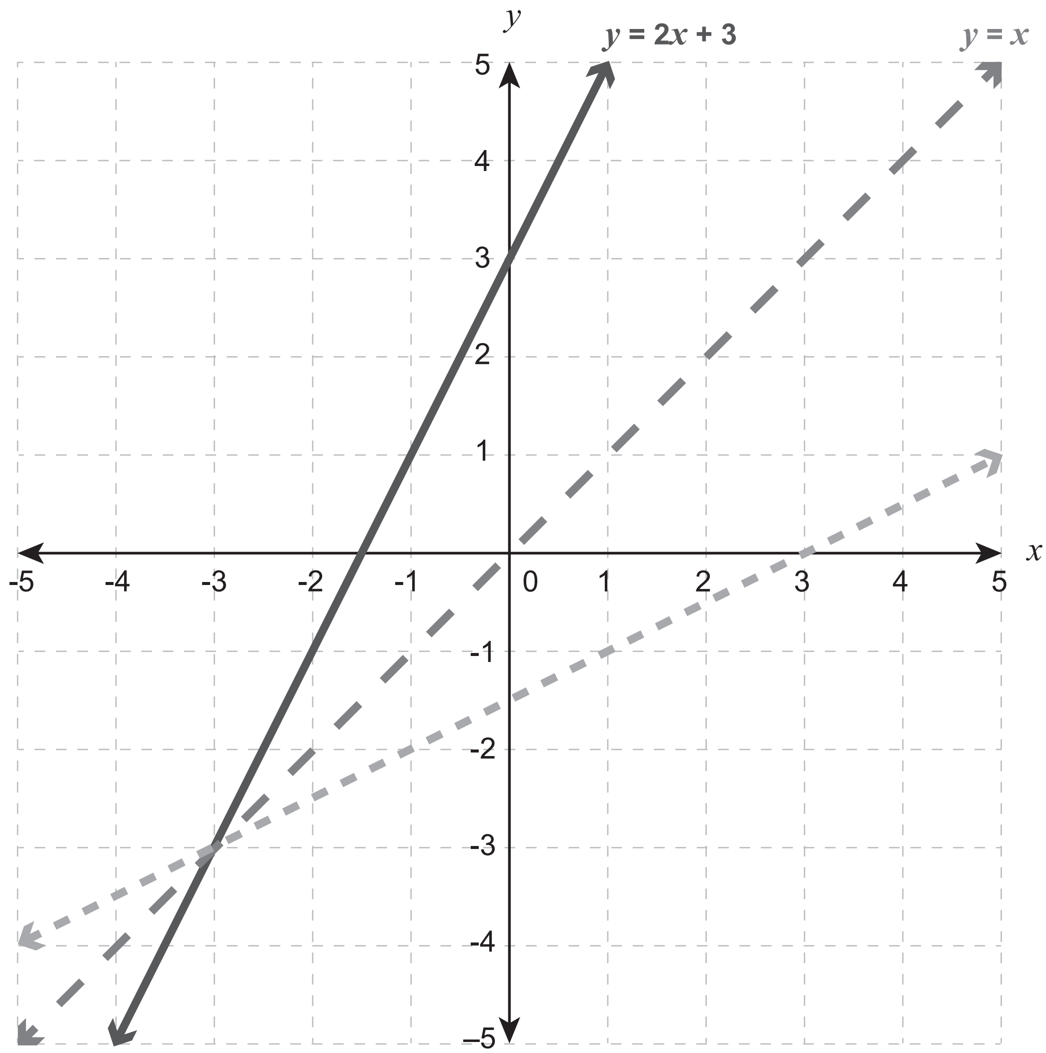
3.1.8 Are the two graphs in QUESTION 3.1.7 functions? Give a reason or reasons for your answer.

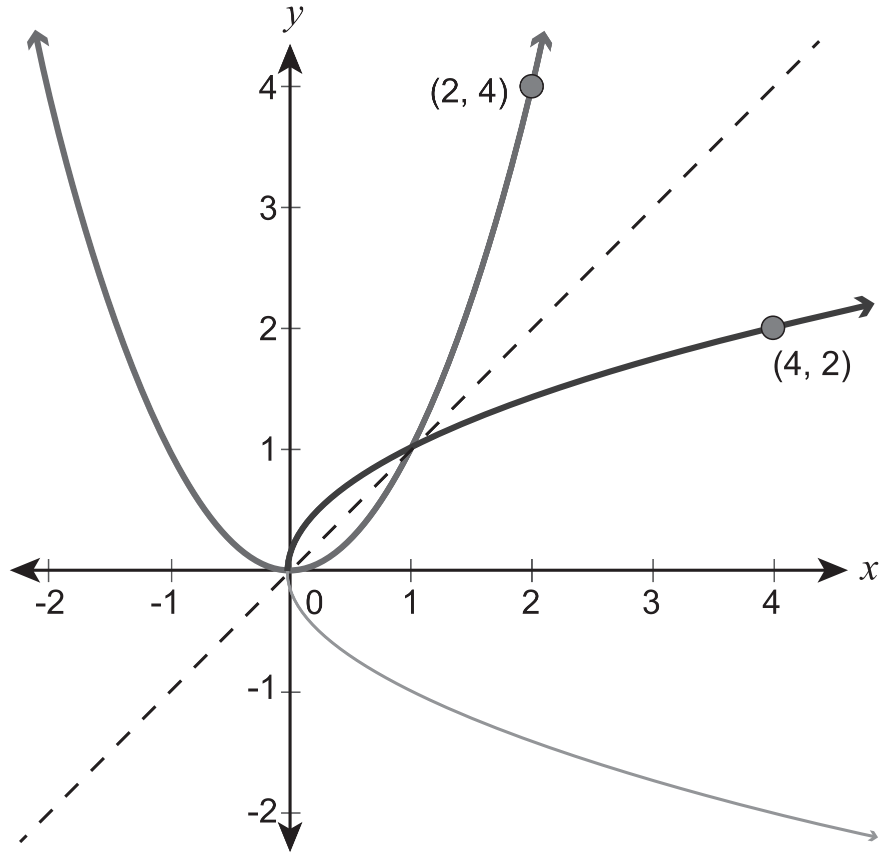
__________________________________________________ (2)

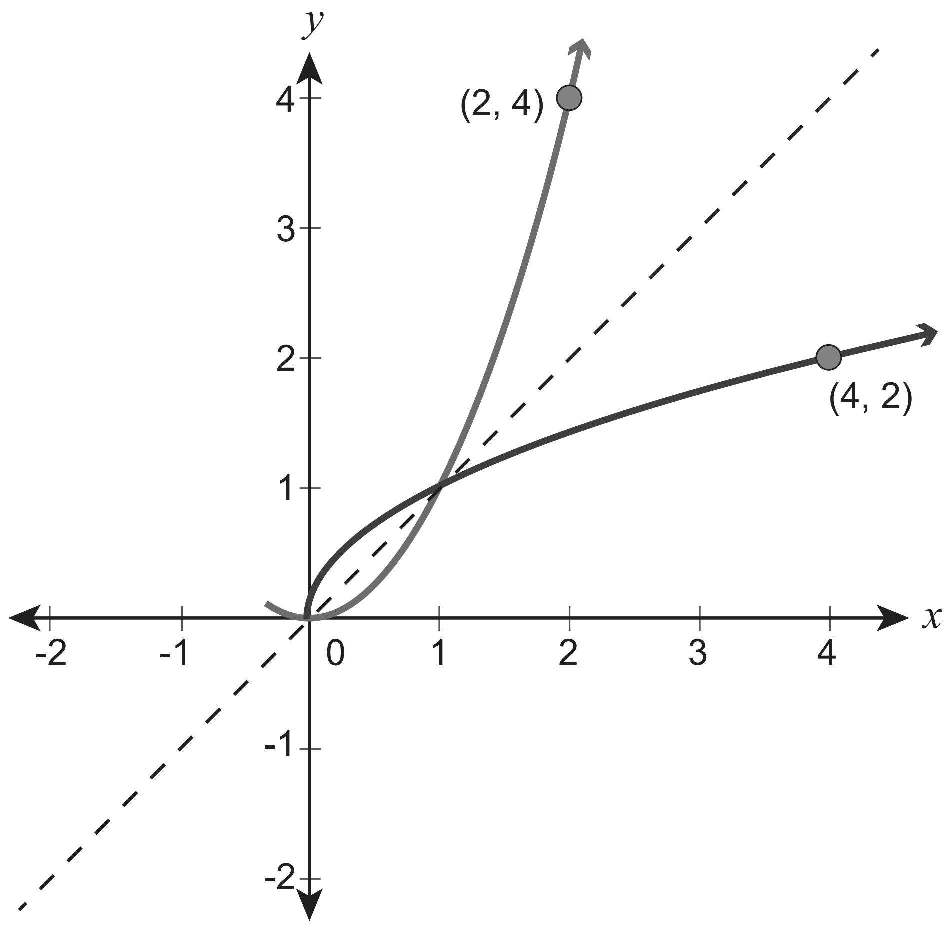
**TOTAL: 50**

# RESOURCE 4

## LESSON 3




y
x
(2, 4)
(4, 2)
4
3
2
1
-1
-2
-2
-1
0
1
2
3
4


y
x
(2, 4)
(4, 2)
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-1
-2
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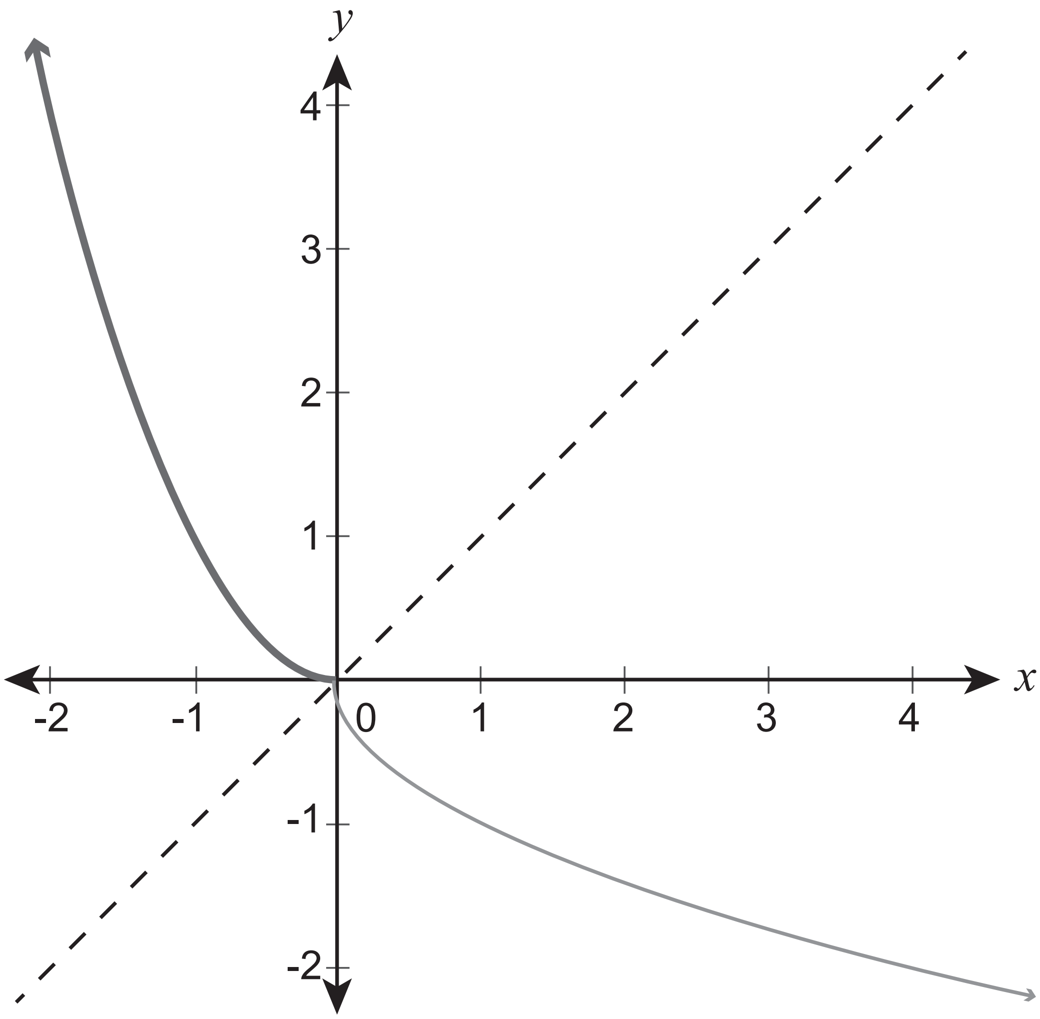
-1
0
1
2
3
4


y
4
3
2
1
-1
-2
x
-2
-1
0
1
2
3
4

# RESOURCE 5

## LESSON 5

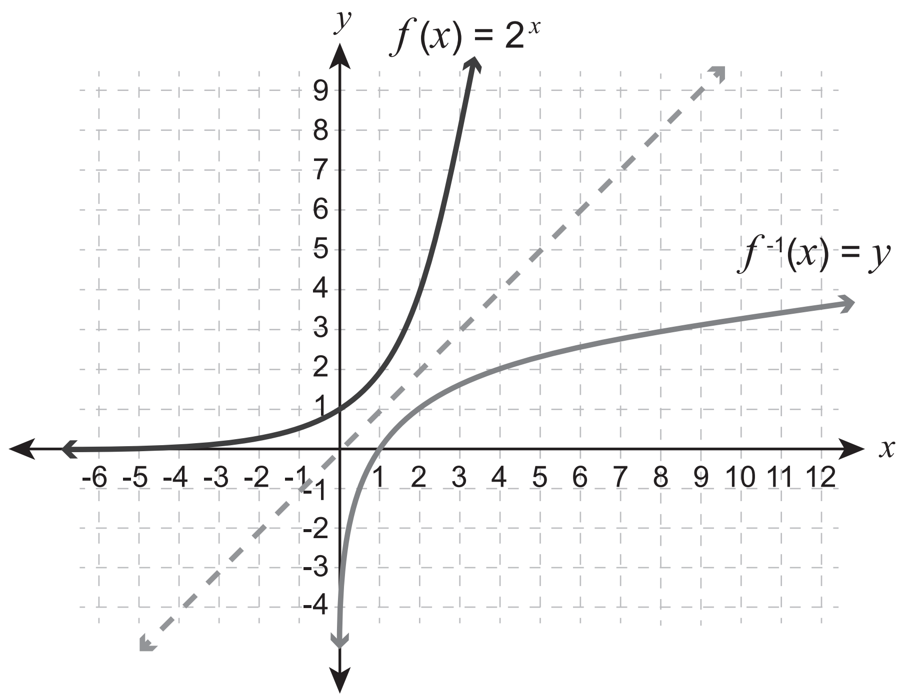
10 x Larger

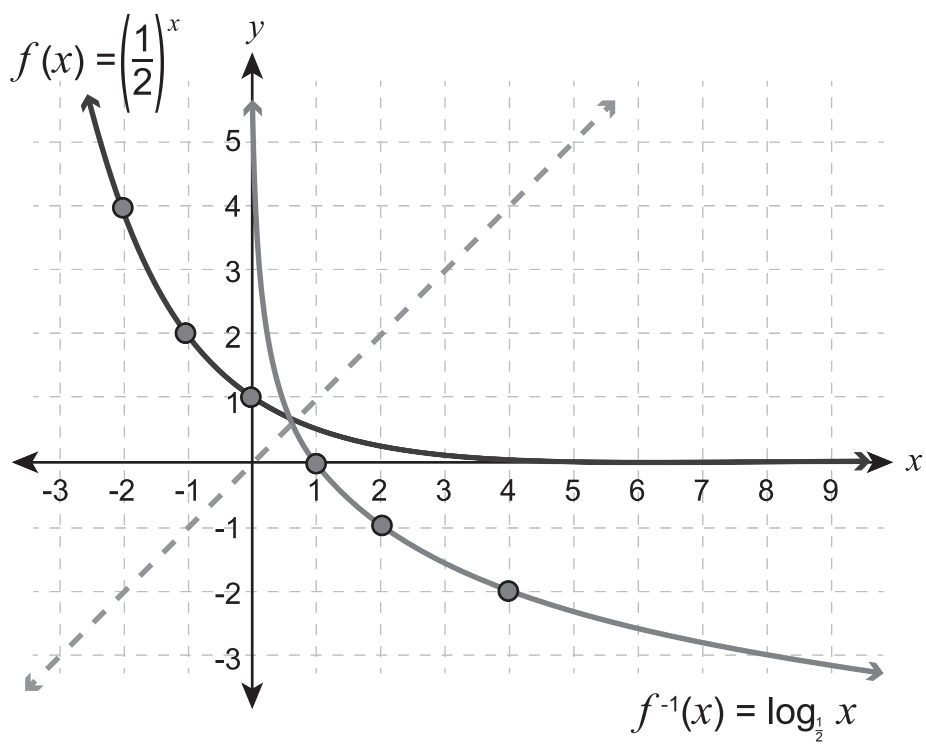
10 x Smaller

| Number | How Many 10s | Base-10 Logarithm | |
|---|---|---|---|
| etc. | | | |
| 1000 | $1 \times 10 \times 10 \times 10$ | $\log_{10}(1000)$ | = 3 |
| 100 | $1 \times 10 \times 10$ | $\log_{10}(100)$ | = 2 |
| 10 | $1 \times 10$ | $\log_{10}(10)$ | = 1 |
| 1 | 1 | $\log_{10}(1)$ | = 0 |
| 0.1 | $1 \div 10$ | $\log_{10}(0.1)$ | = –1 |
| 0.01 | $1 \div 10 \div 10$ | $\log_{10}(0.01)$ | = –2 |
| 0.001 | $1 \div 10 \div 10 \div 10$ | $\log_{10}(0.001)$ | = –3 |
| etc. | | | |

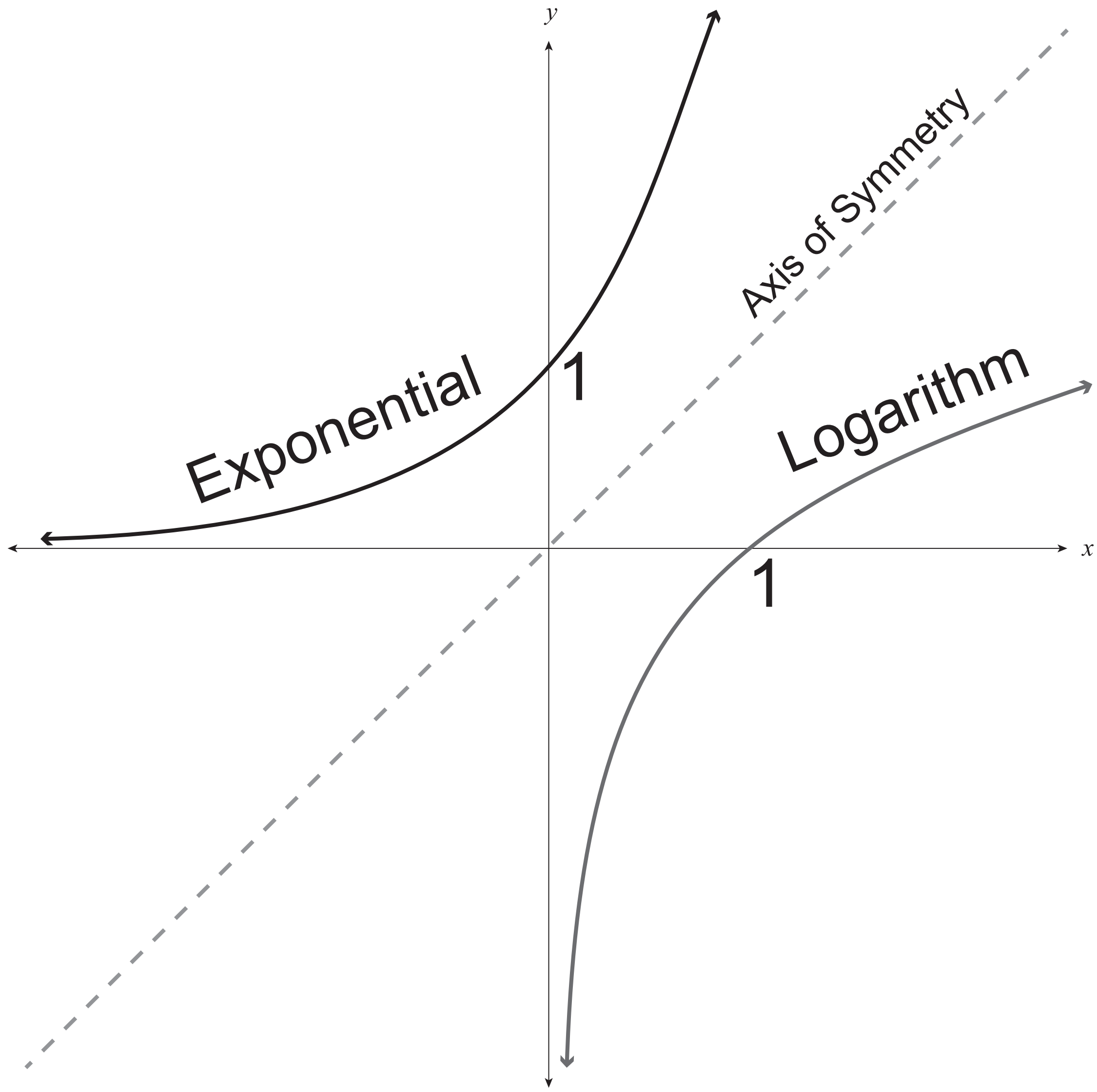
# RESOURCE 6

## LESSON 6



Display on the wall of your classroom:



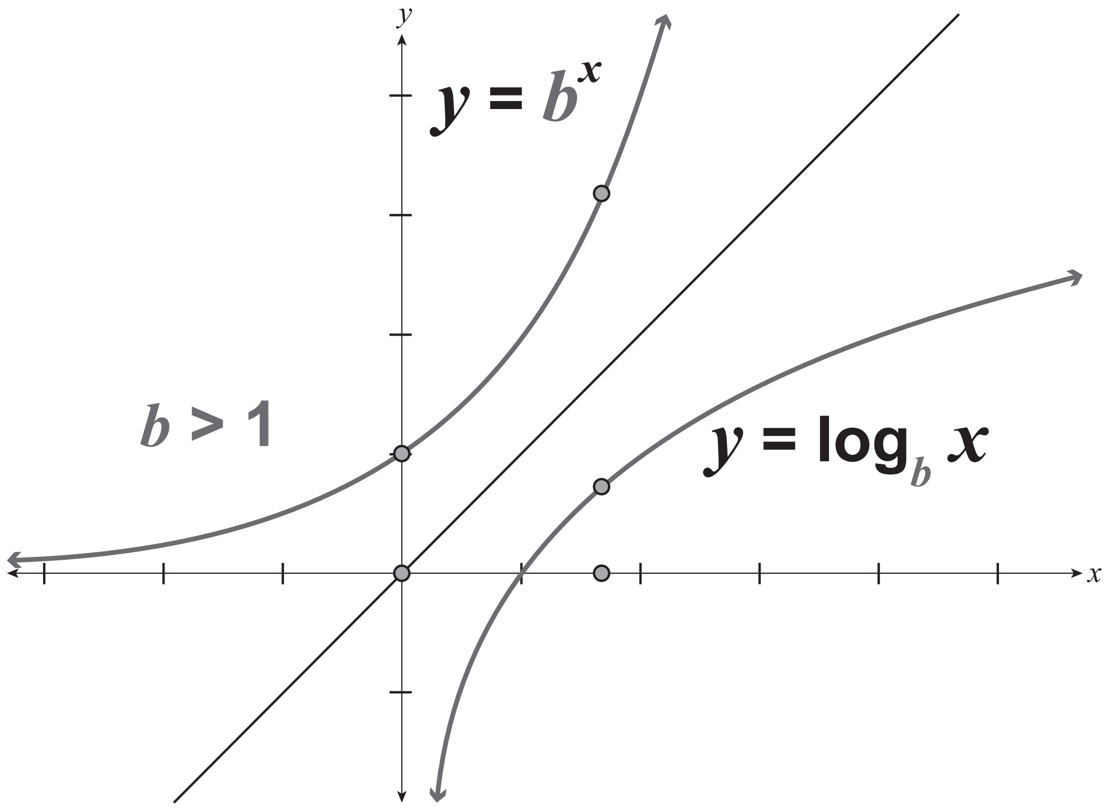



Figure 1, $b > 1$

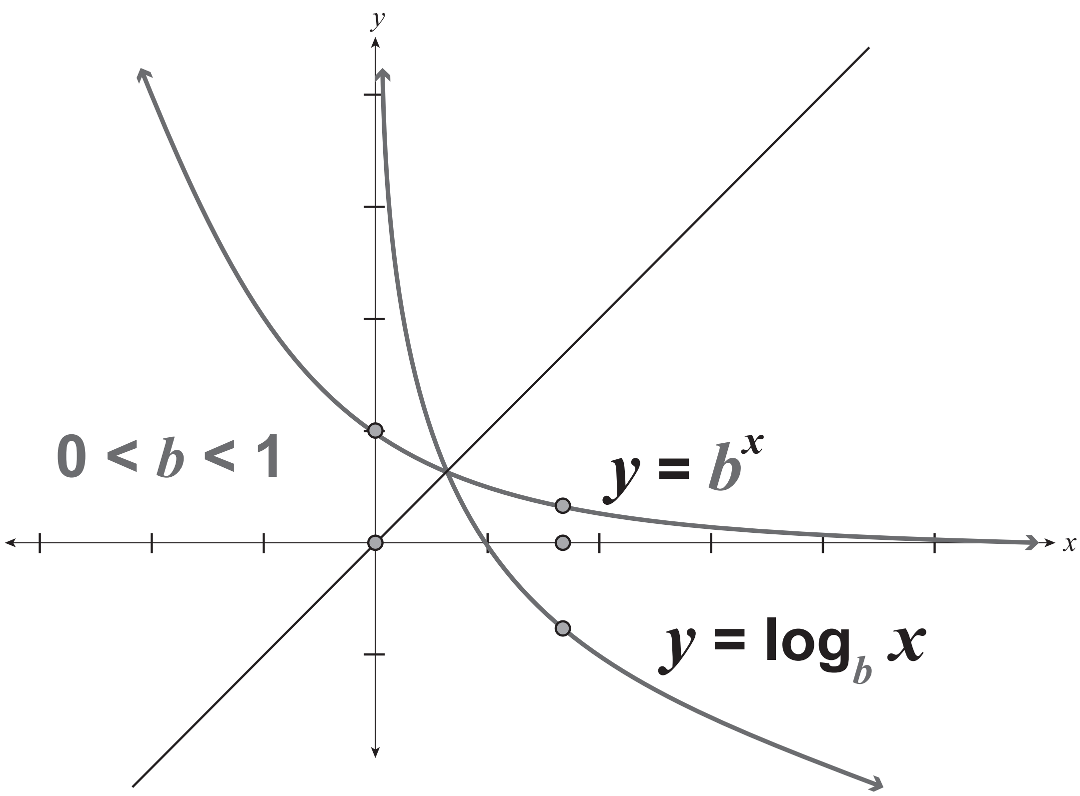



Figure 2, $0 < b < 1$

# FINANCE, GROWTH AND DECAY

## RESOURCE 7

### LESSON 2

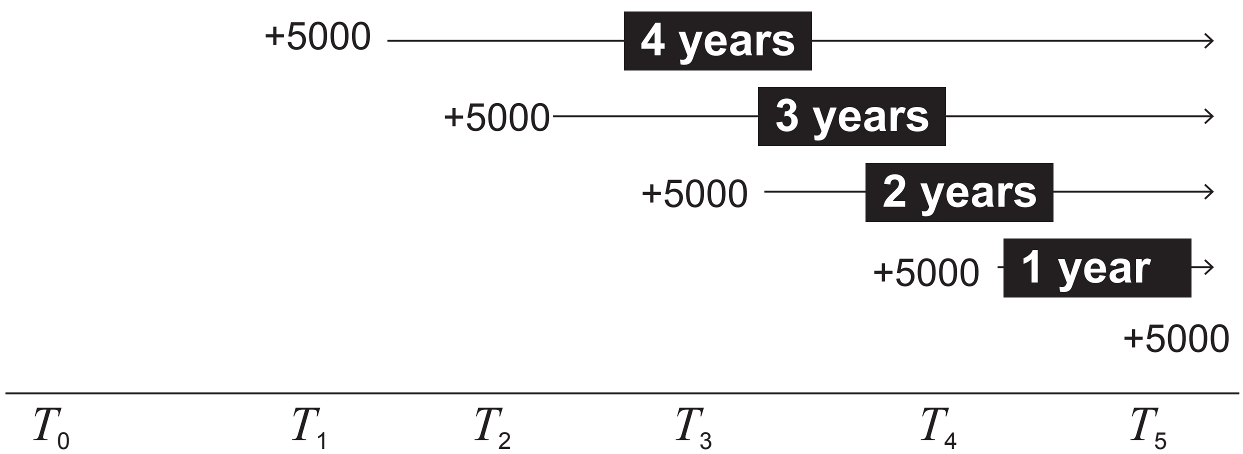
Regular payment

number of payments

$$F_v = \frac{x\,[(1+i)^n - 1]}{i}$$

Future value

interest



# RESOURCE 8

## LESSON 3

Regular payment  number of payments

$$P_v = \frac{x\,[1 - (1 + i)^{-n}]}{i}$$

Present value  interest rate

# TRIGONOMETRY

## RESOURCE 9

### LESSON 1

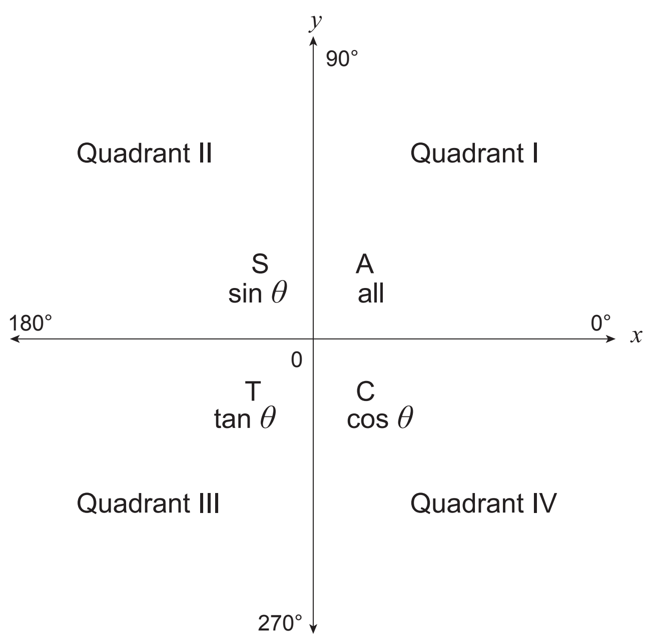
## TRIGONOMETRY

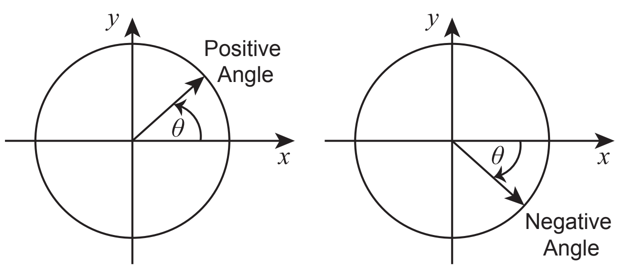
### GRADE 10 AND 11 WORK

- **Basic Ratios**

| | | |
|---|---|---|
| $\sin\theta = \dfrac{opp}{hyp} = \dfrac{y}{r}$ | $\cos\theta = \dfrac{adj}{hyp} = \dfrac{x}{r}$ | $\tan\theta = \dfrac{opp}{adj} = \dfrac{y}{x}$ |

- **Co-ratios, reductions and negative angles (CAST)**

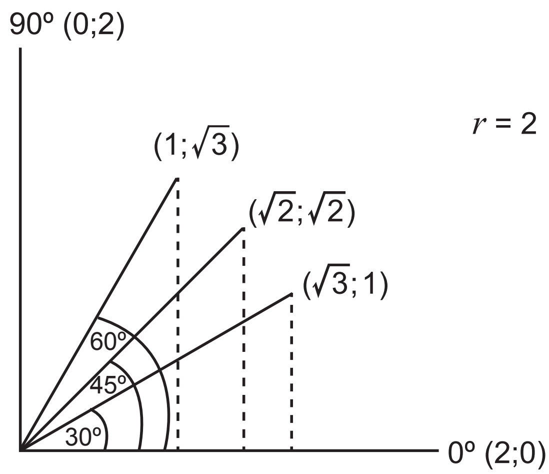



NOTE: Adding 360° or subtracting 360° from an angle does not change where the angle will lie

| Co-ratios<br>(co – complement - 90º) | Negative angles |
|---|---|
| $\sin(90^\circ - \theta) = \cos\theta$<br>$\cos(90^\circ - \theta) = \sin\theta$<br>$\sin(90^\circ + \theta) = \cos\theta$<br>$\cos(90^\circ + \theta) = -\sin\theta$ | $\cos(-\theta) = \cos\theta$<br>$\sin(-\theta) = -\sin\theta$<br>$\tan(-\theta) = -\tan\theta$ |

- **Special angles**




- **Identities**

| | |
|---|---|
| $sin^2\theta + cos^2\theta = 1$ | $tan\ \theta = \dfrac{\sin\theta}{\cos\theta}$ |
| As well as: | |
| $sin^2\theta = 1 - cos^2\theta$ OR $cos^2\theta = 1 - sin^2\theta$<br>And both versions could factorise using difference of 2 squares | $\dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$ |

### REDUCTIONS

Recognised by: The instruction 'simplify' and usually many trig functions and angles within one question. Most often a fraction.

Method:

- Any negative angles – add 360º until its positive (unless between 0º & – 90º then its quicker to use 4th quadrant immediately)
- Any angles bigger than 360º, subtract 360º until the angle lies between 0º & 360º
- Check which quadrant an angle lies in
- Reduce to 180º + /180º– /360º – depending on quadrant
- Decide if its positive or negative

- Reduce to acute angle only (could be a variable or special angle at this stage)
- Remember to look out for co-ratios. If there is a '90º', the trig function will always change.
  sin $\leftrightarrow$ cos
- Keep a look out for special angles

Examples

Simplify:

1. $\dfrac{\cos 140^0 . \sin(90^0 - \theta)}{\sin 410^0 . \cos(-\theta)}$

2. $\dfrac{\cos(360^0 - x) . \sin(360^0 - x) . \cos(90^0 - x)}{\sin^2(90^0 - x) . \sin(180^0 + x)}$

### PYTHAGORAS QUESTIONS

(ALWAYS use a diagram if one is not given)

#### TYPE 1

Recognised by: No calculator instruction & sin/cos/tan of an angle as well as an extra piece of information regarding the size of the angle. These questions DO NOT require you to find the size of the angle.

Tips:

- Ensure you know the relationship with $x, y, r$ and all trig ratios!
- Don't even consider the 'question' until all the ground work has been done.

Method:

- Using BOTH pieces of info, decide which quadrant you need to work in
- Make a sketch, drawing the triangle in the correct quadrant.
- Fill in the two known sides from the given info
- Use Pythagoras to find the third side
- Summarise the info you now know regarding what $x$, $y$ and $r$ are all equal to (Be careful of signs here!)
- Use this information to complete the question using substitution.

Example

Given $sin\,\theta = \dfrac{-9}{41}$ and $cos\,\theta < 0$, determine $\dfrac{\cos\theta}{\sin\theta}$

### TYPE 2

Recognised by: No calculator instruction. sin/cos/tan of an angle is equal to a variable

Method:

- Make the variable into a fraction (put it over 1)
- Draw a right-angled triangle. Fill in the angle and the two know sides from the fraction
- Fill in the other angle (using angles of a triangle) – (this is sometimes used so worth putting in at the beginning)
- Use Pythagoras to find the third side in terms of the variable (expect a root sign in the answer)
- Use the information found to substitute and complete the questions.
    - ALL questions will relate to the angles in your triangle
    - Watch out for reductions

Example

1. If $\tan 36° = p$, express the following in terms of $p$:

   a) $tan\ 144°$ b) $sin\ 306°$ c) $cos\ 234°$

## GENERAL SOLUTIONS (EQUATIONS)

Recognised by: 'Solve the following….' or 'Find the general solution….'

### TYPE 1

Method:

- Make the trig function the subject of the formula
- Use the 2nd function on the calculator: (shift ; trig function ; ratio) to find the reference angle
- Note whether the function is positive or negative
- Choose the quadrants accordingly and find the general solutions according to the quadrants
- Use the appropriate reductions to represent angles in the chosen quadrants.
- Use $k$ to show that it is a general solution and if required, substitute integers to find specific solutions.

Example

Find the general solution: $2\,sin(x + 10°) = cos\,333{,}5°$ if $x \in (0°;180°)$

### TYPE 2

Method:

- If the angles are identical, divide both sides by cos of the angle to change it into a tan equation and solve as in previous type.

Example: $\sin(\alpha + 20°) = \cos(\alpha + 20°)$

### TYPE 3

Method:

- Notice the angles are not identical.
- Use co-ratios to get same trig function on each side
- Treat left hand unknown as normal (making it the unknown) and the right hand unknown as the reference angle.
- Create the general solution as usual.

Example: Determine the general solution to $\cos(2\theta + 30°) = \sin\theta$

General tips for equations:

- Quadratics are common: look for factorising opportunities
- Once factorised, two equations will be created to solve as usual.

### IDENTITIES

Recognised by: Instruction is ‘Prove that….’ Or ‘Prove the identity…’

Method

All of these steps/tips are merely a guide. The more you practise, the easier it will become to ‘see’ your options and know what is required.

- Work with one side at a time (LHS or RHS). Start with the side that looks like you can manipulate more
- Change $\tan\theta$ to $\frac{\sin\theta}{\cos\theta}$
- If there is any addition or subtraction of fractions, find LCD and get one term.
- Look for factorising opportunities. (HCF/Diff of 2 squares/trinomial)
- Watch out for “1”: It may be useful to change it to $\sin^2\theta + \cos^2\theta$
- Keep referring back to the side you are trying to prove to make sure you are heading in the right direction

Examples:

Prove the following identities:

1. $$\frac{(\tan^2\theta - \sin^2\theta)\left(\frac{\cos^2\theta}{\sin^2\theta} + 1\right)}{\tan^2\theta} = 1$$

2. $$\frac{1}{1 - \sin\theta} + \frac{1}{1 + \sin\theta} = \frac{2}{\cos^2\theta}$$

3. $$\frac{\sin\alpha(1 + \cos\alpha)}{1 - \cos^2\alpha} + \tan\alpha = \frac{1 + \cos\alpha}{\sin\alpha.\cos\alpha}$$

Invalid identities:

Working with invalid identities is directly linked to equations. Any identity is invalid for some values if tan is involved (asymptotes for the function) or if there is a fraction involved (zero cannot be in the denominator).

Method:

- If tan: the identity is always invalid for $\theta = 90º + k.180º$ (k ∈ Z)
- The denominator of a fraction is always invalid for all values that make it equal to zero. Form an equation (denominator = 0) and solve.

Example:

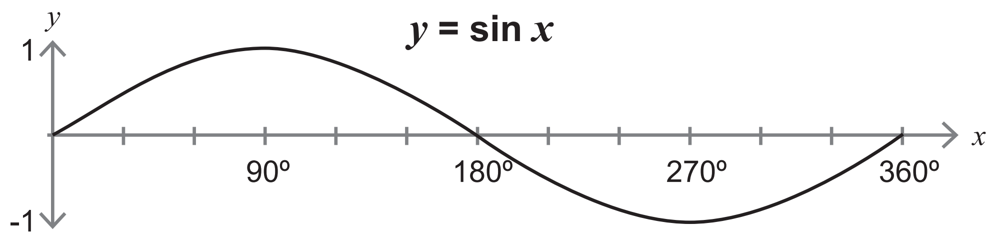
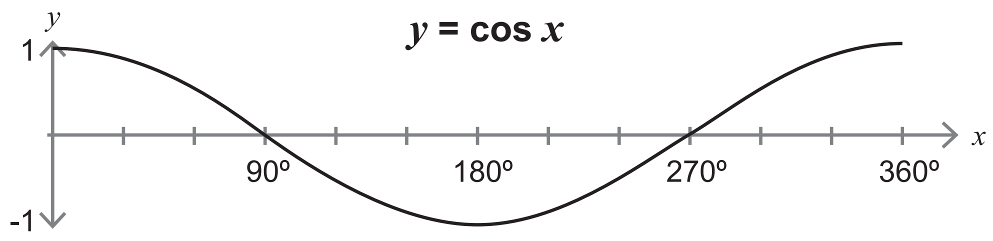
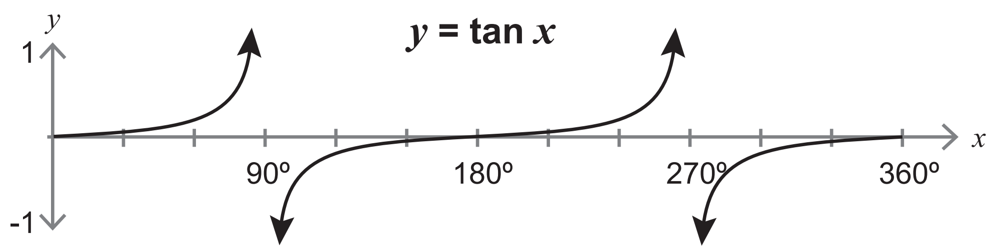
For which values is the identity $\frac{\sin\alpha(1 + \cos\alpha)}{1 - \cos^2\alpha} + \tan\alpha = \frac{1 + \cos\alpha}{\sin\alpha.\cos\alpha}$ invalid?

### FUNCTIONS

Know the three basic functions well! Do not rely on your calculator to draw a trigonometric function.

Ensure you know which variables affect which transformations (horizontal shift, vertical shift, period change, amplitude change).

| | | |
|---|---|---|
| a | Amplitude | 'Stretches' or 'squashes' the graph<br>Amplitude = $\frac{1}{2}$ height<br>A negative amplitude will turn the graph over (reflection of original) |
| b | Period | $\frac{\textit{original period}}{b} = \textit{new period}$<br>'$b$' tells you how many graphs you will see in the original period |
| p | Vertical shift | Number of units ($y$-axis) a graph shifts up or down |
| q | Horizontal shift | Number of degrees ($x$-axis) a graph shifts left or right<br>Remember: '–' means shift right and '+' means shift left |

| | |
|---|---|
| Sine graph | A function of the form $y = sin\ x$ where $x \in (0º;360º)$<br> |
| Cosine graph | A function of the form $y = cos\ x$ where $x \in (0º;360º)$<br> |
| Tangent graph | A function of the form $y = tan\ x$ where $x \in (0º;360º)$<br> |

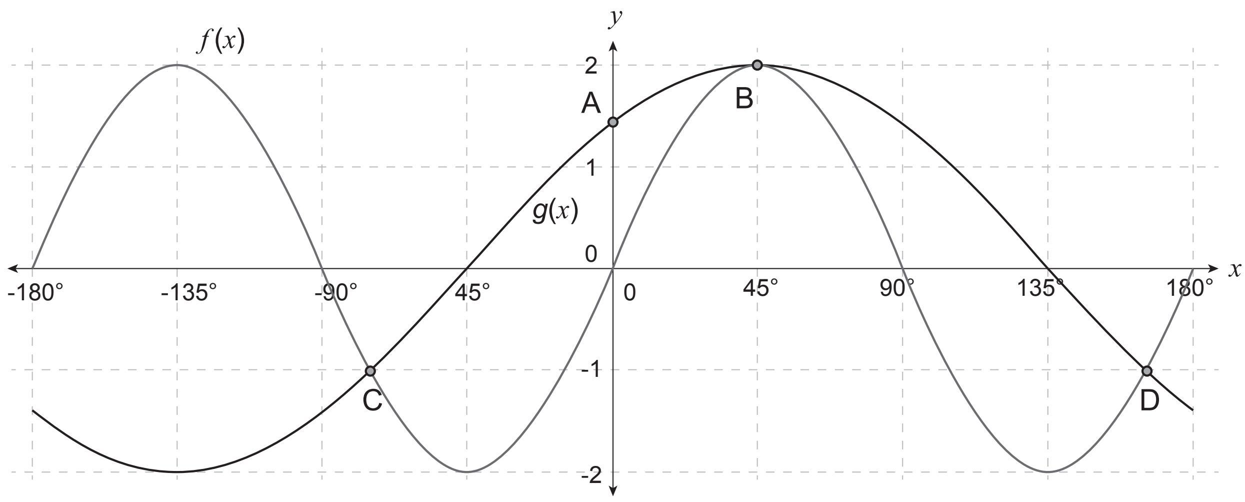
As well as these basics, you also need to be able to answer questions about domain, range, distance between graphs as well as finding values of $x$ given certain conditions (eg $f(x) \leq g(x)$)

Method

- Add whatever you can to these diagrams to improve the accuracy.
- If you are required to draw a graph, it is often a good idea to draw the basic one, then make the alterations according to any shifts, period changes or amplitude changes.
- To find points of intersection, make the equations equal and solve. Keep in mind the domain given, to find the specific solutions covering the points of intersection.

Example

The sketch shows the graphs of $f(x) = a \sin kx$ and $g(x) = a\cos(x - p)$ for $x \in [-180°;180°]$




a. Write down the range of $g$.

b. Write down the period of $f$.

c. Determine the values of $a$ ;$k$ and $p$.

d. Determine the coordinates of A, the y-intercept of $g$.

e. Write down the coordinates of B

f. If the co-ordinates of C and D are (-75°;-1) and (165°;-1) respectively, write down the values of $x \in$ [-180°;180°] for which f < $g$

g. The graph of $g$ is shifted 45° to the right and given a new name $p$. Write the equation of $p$ in two ways, using different ratios in each equation.

# TRIGONOMETRY – SOLUTIONS

## REDUCTIONS

Examples

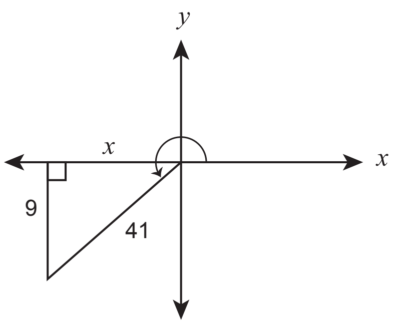
| | |
|---|---|
| $\dfrac{\cos 140^0 . \sin(90^0 - \theta)}{\sin 410^0 . \cos(-\theta)}$<br>$= \dfrac{\cos(180^0 - 40^0) . -\cos\theta}{\sin 50^0 . \cos\theta}$<br>$= \dfrac{-\cos 40^0 . -\cos\theta}{\sin 50^0 . \cos\theta}$<br>$= 1$ | $\dfrac{\cos(360^0 - x) . \sin(360^0 - x) . \cos(90^0 - x)}{\sin^2(90^0 - x) . \sin(180^0 + x)}$<br>$= \dfrac{\cos x . -\sin x . \sin x}{\cos^2 x . \sin x}$<br>$= \dfrac{\sin x}{\cos x}$<br>$= \tan x$ |

## PYTHAGORAS QUESTIONS

### TYPE 1

Example

Given $sin\ \theta = \dfrac{-9}{41}$ and $cos\ \theta < 0$, determine $\dfrac{\cos\theta}{\sin\theta}$

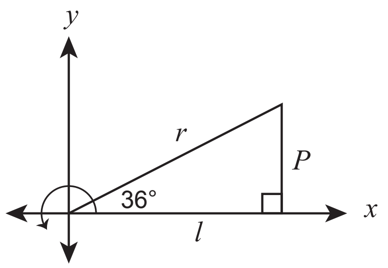
| | | |
|---|---|---|
|  | $x^2 + y^2 = r^2$<br>$x^2 + 9^2 = 41^2$<br>$x^2 = 41^2 - 9^2$<br>$x^2 = 1600$<br>$x = 40$<br>$\therefore\ x = -40;\ y = -9;\ r = 41$ | $\dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{-40}{41}}{\frac{-9}{41}}$<br>$= \dfrac{-40}{41} \times \dfrac{41}{-9}$<br>$= \dfrac{40}{9}$ |

### TYPE 2

Example

1. If tan 36º = $p$, express the following in terms of $p$:

a) $tan$ 144º  b) $sin$ 306º  c) $cos$ 234º




$$r^2 = 1^2 + p^2$$

$$r = \sqrt{1+p^2}$$

$$\therefore x = 1\ ;\ y = p\ ;\ r = \sqrt{1+p^2}$$

a) tan 144º
 = tan (180º – 36º )
 = – tan 36º
 = – p

b) sin 306º
 = sin (360º – 54º)
 = – sin 54º
 = – cos 36º
 $= -\dfrac{p}{\sqrt{1+p^2}}$

c) cos 234º
 = cos (180º + 54º)
 = – cos 54º
 = – sin 36º
 $= -\dfrac{1}{\sqrt{1+p^2}}$

### GENERAL SOLUTIONS (EQUATIONS)

### TYPE 1

Example

Find the general solution: 2 sin ($x$ + 10º ) = cos 333,5º if $x \in$ (0º;180º )

2 sin ($x$ + 10º ) = cos 333,5º
2 sin ($x$ + 10º ) = 0,895
sin ($x$ + 10º ) = 0,447
Ref angle (RA) = 26,55º
Positive ratio ∴ Quadrant 1 and 2

Quad 1:
$x$ + 10º = $RA$ + $k$.360º
$x$ + 10º = 26,55º + $k$.360º
$x$ = 16,55º + $k$.360º

Quad 2:
$x$ + 10º = (180º – $RA$) + $k$.360º
$x$ + 10º = (180º – 26,55º) + $k$.360º
$x$ = 143,45º + $k$.360º

$k \in Z$

### TYPE 2

Example: sin (α + 20º ) = cos (α + 20º )

$$\sin(\alpha + 20^\circ) = \cos(\alpha + 20^\circ)$$
$$\frac{\sin(\alpha + 20^0)}{\cos(\alpha + 20^0)} = \frac{\cos(\alpha + 20^0)}{\cos(\alpha + 20^0)}$$
$$\tan(\alpha + 20^\circ) = 1$$
$$RA = 45^\circ$$

Quad 1:
$x = RA + k.180^\circ$
$x = 45^\circ + k.180^\circ$

Quad 3:
$x = (180^\circ - RA) + k.180^\circ$
$x = (180^\circ - 45^\circ) + k.180^\circ$
$x = 225^\circ + k.180^\circ$

$k \in Z$

### TYPE 3

Example: Determine the general solution to cos (2θ + 30º ) = sin $\theta$

$$\cos(2\theta + 30^\circ) = \sin\theta$$
$$\cos(2\theta + 30^\circ) = \cos(90^\circ - \theta)$$

Quad 1:
$2\theta + 30^\circ = 90^\circ - \theta + k.360^\circ$
$3\theta = 60^\circ + k.360^\circ$
$\theta = 20^\circ + k.180^\circ$

Quad 4:
$2\theta + 30^\circ = 360^\circ - (90^\circ - \theta) + k.360^\circ$
$2\theta + 30^\circ = 270^\circ + \theta + k.360^\circ$
$\theta = 240^\circ + k.360^\circ$

$k \in Z$

### IDENTITIES

Examples:

Prove the following identities:

1. $\dfrac{(\tan^2\theta \sin^2\theta)\left(\dfrac{\cos^2\theta}{\sin^2\theta} + 1\right)}{\tan^2\theta} = 1$

2. $\dfrac{1}{1 - \sin\theta)} + \dfrac{1}{1 + \sin\theta} = \dfrac{2}{\cos^2\theta}$

3. $\dfrac{\sin\alpha(1 + \cos\alpha)}{1 - \cos^2\alpha} + \tan\alpha = \dfrac{1 + \cos\alpha}{\sin\alpha.\cos\alpha}$

| | |
|---|---|
| 1. LHS = $\dfrac{\left(\dfrac{\sin^2\theta}{\cos^2\theta}-\dfrac{\sin^2\theta}{1}\right)\left(\dfrac{\cos^2\theta}{\sin^2\theta}-1\right)}{\dfrac{\sin^2\theta}{\cos^2\theta}}$<br>$=\left(\dfrac{\sin^2\theta-\sin^2\theta.\cos^2\theta}{\cos^2\theta}\right)\left(\dfrac{\cos^2\theta+\sin^2\theta}{\sin^2\theta}\right)\left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)$<br>$=\left(\dfrac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}\right)\left(\dfrac{1}{\sin^2\theta}\right)\left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)$<br>$=\dfrac{\sin^2\theta.\sin^2\theta.\cos^2\theta}{\cos^2\theta.\sin^2\theta.\sin^2\theta}$<br>= 1 = RHS | 2. LHS = $\dfrac{1}{1-\sin\theta}+\dfrac{1}{1+\sin\theta}$<br>$=\dfrac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$<br>$=\dfrac{2}{1-\sin^2\theta}$<br>$=\dfrac{2}{\cos^2\theta}$ =RHS<br>3. LHS = $\dfrac{\sin\alpha(1+\cos\alpha)}{1-\cos 2\alpha}+\tan\alpha$<br>$=\dfrac{\sin\alpha(1+\cos\alpha)}{\sin^2\alpha}+\dfrac{\sin\alpha}{\cos\alpha}$<br>$=\dfrac{1+\cos\alpha}{\sin\alpha}+\dfrac{\sin\alpha}{\cos\alpha}$<br>$=\dfrac{\cos\alpha(1+\cos\alpha)+\sin^2\alpha}{\sin\alpha.\cos\alpha}$<br>$=\dfrac{\cos\alpha+\cos^2\alpha+\sin^2\alpha}{\sin\alpha.\cos\alpha}$<br>$=\dfrac{1+\cos\alpha}{\sin\alpha.\cos\alpha}$ = RHS |

Invalid identities:

For which values is the identity $\dfrac{\sin\alpha(1+\cos\alpha)}{1-\cos^2\alpha}+\tan\alpha=\dfrac{1+\cos\alpha}{\sin\alpha.\cos\alpha}$ invalid?

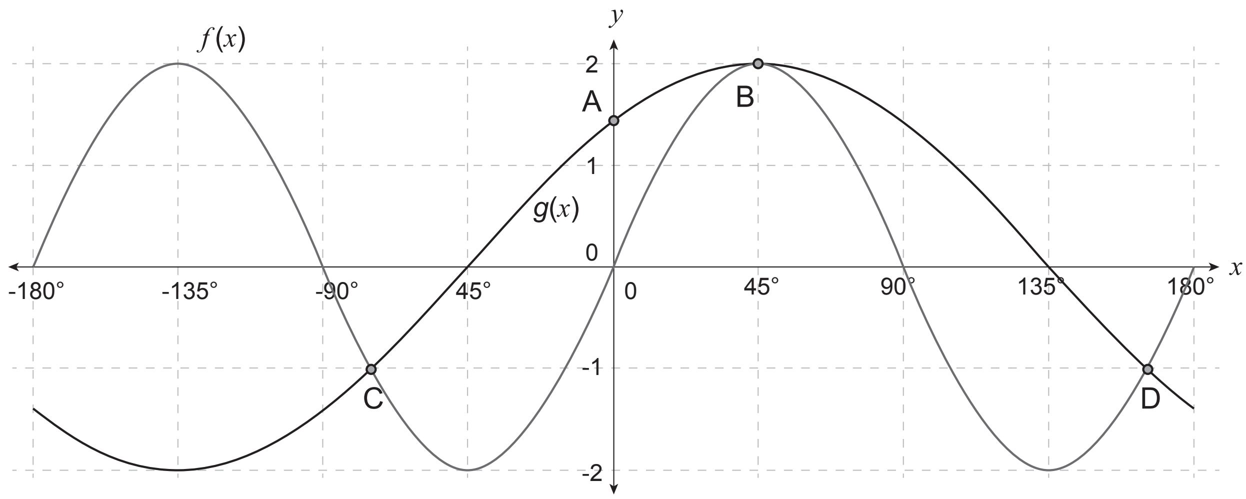
| | |
|---|---|
| $1-\cos^2\alpha=0$<br>$(1+\cos\alpha)(1-\cos\alpha)=0$<br>$1+\cos\alpha=0$ or $1-\cos\alpha=0$<br>$\cos\alpha=-1$ / $\cos\alpha=1$<br>$RA=0°$ / $RA=0°$<br>Quad 2: $\alpha=180°-0°+k.360°$; $\alpha=180°+k.360°$<br>Quad 1: $\alpha=0°+k.360°$; $\alpha=k.360°$<br>Quad 3: $\alpha=180°+0°+k.360°$; **$\alpha=180°+k.360°$**<br>Quad 4: $\alpha=360°-0°+k.360°$; $\alpha=360°+k.360°$ | $\sin\alpha.\cos\alpha=0$<br>$\sin\alpha=0$ or $\cos\alpha=0$<br>$RA=0°$ / $RA=90°$<br>Quad 1: $\alpha=0°+k.360°$; $\alpha=k.360°$<br>Quad 1: $\alpha=0°+k.360°$; $\alpha=k.360°$<br>Quad 2: $\alpha=180°-0°+k.360°$; $\alpha=180°+k.360°$<br>Quad 4: $\alpha=360°-0°+k.360°$; $\alpha=360°+k.360°$ |
| $\tan\alpha$ : **$\alpha=90°+k.180°$** | |

Note: although this example of an invalid identity is quite cumbersome, it is useful when revising as it combines many different concepts (as well as practicing solving equations). Remind learners to look for a summary of all the possible values that make the identity invalid. These were highlighted in bold.

**FUNCTIONS**

Example

The sketch shows the graphs of f($x$)=a sin $kx$ and g(x)=acos(x - p) for $x \in [-180°;180°]$




a. Write down the range of *g*.

b. Write down the period of $f$.

c. Determine the values of $a$ ; $k$ and $p$.

d. Determine the coordinates of A, the $y$-intercept of *g*.

e. Write down the coordinates of B

f. If the co-ordinates of C and D are (-75°;-1) and (165°;-1) respectively, write down the values of $x \in [-180°;180°]$ for which $f < g$

g. The graph of *g* is shifted 45° to the right and given a new name $p$. Write the equation of $p$ in two ways, using different ratios in each equation.

| | |
|---|---|
| a | $y \in [-2; 2]$ |
| b | 180º |
| c | $a = 2$ ; $k = 2$ ; $p = 45º$ |
| d | $y = 2\cos(-45º) = \sqrt{2}$ $\therefore A\ (0;\sqrt{2})$ |
| e | $B(45º;2)$ |
| f | $x \in (-75º;165º)$; $x \neq 45º$ |
| g | $p(x) = 2\cos(x - 90º)$ or $p(x) = 2\sin x$ |

## RESOURCE 10

### LESSON 2: INVESTIGATION

# INVESTIGATION – TRIGONOMETRY

NAME: ____________________________

PART 1

Using a scientific calculator, complete the following table. Write your answer to THREE decimal places. Answers only are acceptable.

| Let $A$ = 20º and $B$ = 10º | |
|---|---|
| cos $(A + B)$ | |
| cos $A$ + cos $B$ | |
| cos $(A - B)$ | |
| cos $A$ – cos $B$ | |
| cos $A$.cos $B$ + sin $A$.sin $B$ | |
| cos $A$.cos $B$ – sin $A$.sin $B$ | |
| Let $A$ = 50º and $B$ = 30º | |
| sin $(A + B)$ | |
| sin $A$ + sin $B$ | |
| sin $(A - B)$ | |
| sin $A$ – sin $B$ | |
| sin $A$.cos $B$ + cos $A$.sin $B$ | |
| sin $A$.cos $B$ – cos $A$.sin $B$ | |

Look carefully at your answers on the right-hand side and make conclusions about the expressions on the left-hand side. Complete the following statements by filling in = or ≠ in the space provided.

| | | | | | |
|---|---|---|---|---|---|
| cos $(A + B)$ | $\square$ | cos $A$ + cos $B$ | sin $(A + B)$ | $\square$ | sin $A$ + sin $B$ |
| cos $(A - B)$ | $\square$ | cos $A$ – cos $B$ | sin $(A - B)$ | $\square$ | sin $A$ – sin $B$ |
| cos $(A + B)$ | $\square$ | cos $A$.cos $B$ – sin $A$.sin $B$ | sin $(A + B)$ | $\square$ | sin $A$.cos $B$ – cos $A$.sin $B$ |
| cos $(A - B)$ | $\square$ | cos $A$.cos $B$ + sin $A$.sin $B$ | sin $(A - B)$ | $\square$ | sin $A$.cos $B$ + cos $A$.sin $B$ |

For those statements that you have concluded are equal, choose two different values for A and B to confirm that you are correct.

| | |
|---|---|
| Statement that seems to be true: | |
| New values tested: $A$ = $B$ = | LHS =<br>RHS = |
| Conclusion: | The statement above is true/false |
| Statement that seems to be true: | |
| New values tested: $A$ = $B$ = | LHS =<br>RHS = |
| Conclusion: | The statement above is true/false |
| Statement that seems to be true: | |
| New values tested: $A$ = $B$ = | LHS =<br>RHS = |
| Conclusion: | The statement above is true/false |
| Statement that seems to be true: | |
| New values tested: $A$ = $B$ = | LHS =<br>RHS = |
| Conclusion: | The statement above is true/false |

Using a scientific calculator, complete the following table. Write your answer to THREE decimal places. Answers only are acceptable.

| | |
|---|---|
| Let $A$ = 20º | |
| sin $2A$ | |
| 2 sin $A$.cos $A$ | |
| Let $A$ = 50º | |
| sin $2A$ | |
| 2 sin $A$.cos $A$ | |

Before completing the next table, remember that $cos^2 10º = (\cos 10º)^2$.
Be careful when using your calculator.

| Let $A$ = 20º | |
|---|---|
| $\cos 2A$ | |
| $2 \cos A - 1$ | |
| $1 - 2\sin^2 A$ | |
| $\cos^2 A - \sin^2 A$ | |
| Let $A$ = 50º | |
| $\cos 2A$ | |
| $2 \cos A - 1$ | |
| $1 - 2\sin^2 A$ | |
| $\cos^2 A - \sin^2 A$ | |

The table above should show that:

| | |
|---|---|
| $\sin 2A = 2 \sin A.\cos A$ | $\cos 2A = 2\cos^2 A - 1$<br>$\cos 2A = 1 - 2\sin^2 A$<br>$\cos^2 A - \sin^2 A$ |

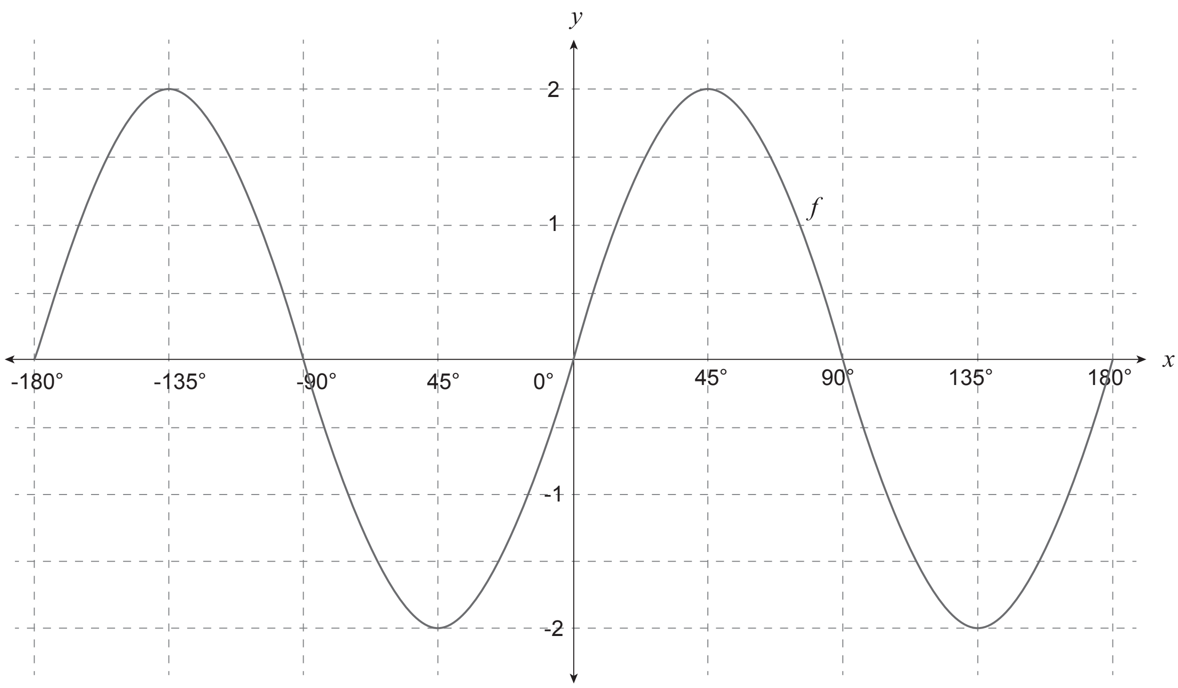
Complete the following:

| |
|---|
| $\sin^2 A + \cos^2 A =$ |
| $\cos(90º - A) =$ |
| $\sin(90º - A) =$ |
| $\cos(90º + A) =$ |
| $\sin(90º + A) =$ |

These will be used in a later lesson to derive all the identities that you have proved to be true.

# RESOURCE 11

## LESSON 7



# RESOURCE 12

## Term 1 Test 1

| QUESTION | DESCRIPTION | MAXIMUM MARK | ACTUAL MARK |
|---|---|---|---|
| 1 | Patterns and Sequences | 23 | |
| 2 | Functions and Graphs | 27 | |
| | TOTAL | 50 | |

**QUESTION 1** **23 MARKS**

1.1 State whether each sequence below is arithmetic, geometric, quadratic, or none of the above.

1.1.1 2; -2; 2; -2; 2; ... (1)

1.1.2 2; 1; 2; 5; 10 ... (1)

1.1.3 1; 2; 4; 9; 16 ... (1)

1.2 Determine the sum of the following series:

1.2.1 24 + 12 + 6 + ... up to 10 terms. (3)

1.2.2 10 + 13 + 16 + 19 + ... + 361 (6)

1.3 The eleventh term of an arithmetic sequence is 40, the sum of the twelfth and twenty-third term is 2. Determine the value of the first term of the sequence. (7)

1.4 Let $p$; $p(p-2)$; $p(p-2)^2$ be the first three terms of a geometric sequence.

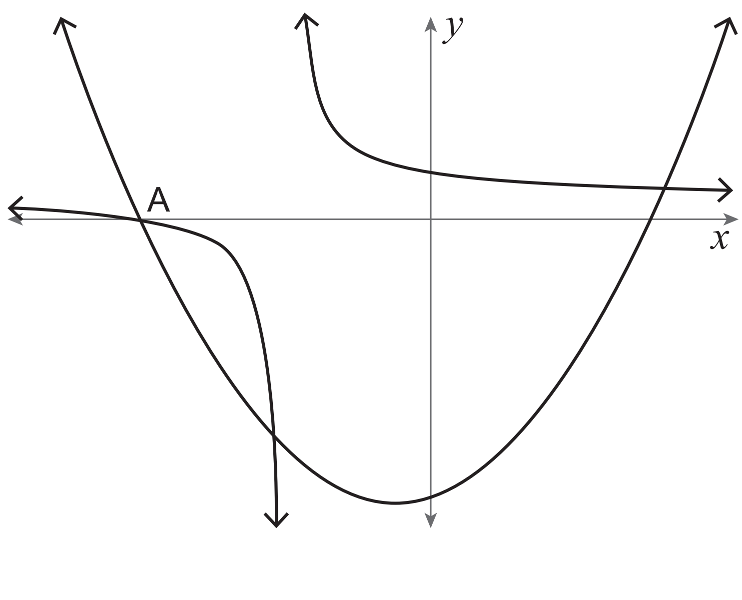
1.4.1 For which values of $p$ will the sequence converge? (2)

1.4.2 If $p = \frac{3}{2}$, determine $S_\infty$. (2)

**QUESTION 2** **27 MARKS**

2.1 Sketched below are the graphs $g(x) = x^2 + x - 12$ and $h(x) = \frac{p}{x+2} + 1$.

- The $y$-intercepts of the graphs are 14 units apart.
- Point A is a point of intersection as well as an $x$-intercept for both graphs.



2.1.1 Is $g(x)$ a function? Give a reason for your answer. (2)

2.1.2 Is the inverse of $g(x)$ a function? Give a reason for your answer. (2)

2.1.3 Determine the coordinates of $A$. (3)

2.1.4 Prove that $p = 2$. (3)

2.1.5 Determine the axis of symmetry with a positive gradient of $h(x)$. (4)

2.2 Given: $f(x) = 3^x - 3$

2.2.1 Write down the equation of $f^{-1}$, the inverse of $f$, in the form $y = ...$ (3)

2.2.2 On the same set of axes, sketch the graphs of $f$ and $f^{-1}$, clearly showing all intercepts with the axes. (5)

2.2.3 Determine the equation of the asymptote for $f^{-1}$. (1)

2.2.4 Write down the domain of both $f$ and $f^{-1}$. (2)

2.2.5 Determine the values of $k$ for which $f(x + 2) = k$ will have no solution. (2)

# RESOURCE 13

## Memorandum Term 1 Test 1

| QUESTION | DESCRIPTION | MAXIMUM MARK | ACTUAL MARK |
|---|---|---|---|
| 1 | Patterns and Sequences | 23 | |
| 2 | Functions and Graphs | 27 | |
| | TOTAL | 50 | |

**QUESTION 1** **23 MARKS**

1.1 State whether each sequence below is arithmetic, geometric, quadratic, or none of the above.

1.1.1 2; -2; 2; -2; 2; ... (1K)
Geometric ✓

1.1.2 2; 1; 2; 5; 10 ... (1K)
Quadratic ✓

1.1.3 1; 2; 4; 9; 16 ... (1K)
None of the above ✓

1.2 Determine the sum of the following series:

1.2.1 24 + 12 + 6 + ... up to 10 terms. (3R)

$S_n = \dfrac{24\left(1 - \frac{1}{2}^{10}\right)}{1 - \frac{1}{2}}$ ✓✓

$S_n = \dfrac{3\,069}{64}$ ✓

1.2.2 10 + 13 + 16 + 19 + ... + 361 (6C)

$T_n = 10 + (n - 1)(3)$ ✓

$T_n = 10 + 3n - 3$

$T_n = 3n + 7$ ✓

$361 = 3n + 7$ ✓

$354 = 3n$

$118 = n$ ✓

$S_n = \dfrac{118}{2}\,[2(10) + (118 - 1)(3)]$ ✓

$S_n = 21\,889$ ✓

1.3 The eleventh term of an arithmetic sequence is 40, the sum of the twelfth and twenty-third term is 2. Determine the value of the first term of the sequence. (7C)

$T_{11} = 40$ ✓ and $T_{12} + T_{23} = 2$ ✓

$a + 10d = 40$ ✓ and $a + 11d + a + 22d = 2$ ✓

$a = -10d + 40$ and $2a + 33d = 2$

$2(-10d + 40) + 33d = 2$ ✓

$-20d + 80 + 33d = 2$

$13d = -78$

$d = -6$ ✓

$a = -10(-6) + 40$

$a = 100$ ✓

1.4 Let $p; p(p-2); p(p-2)^2$ be the first three terms of a geometric sequence.

1.4.1 For which values of $p$ will the sequence converge? (2P)

$-1 < r < 1$

$-1 < p - 2 < 1$ ✓

$1 < p < 3$ ✓

1.4.2 If $p = \frac{3}{2}$, determine $S_\infty$. (2K)

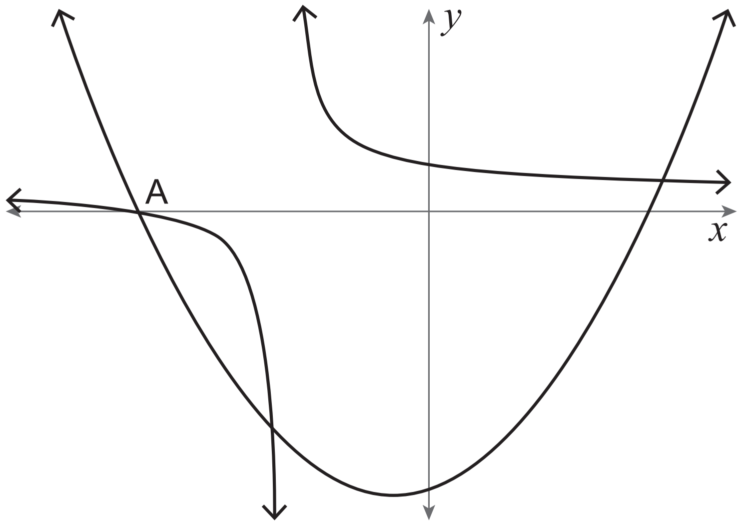
$S_\infty = \dfrac{\frac{3}{2}}{1 + \frac{1}{2}}$ ✓

$S_\infty = 1$ ✓

**QUESTION 2** **27 MARKS**

2.1 Sketched below are the graphs $g(x) = x^2 + x - 12$ and $h(x) = \dfrac{p}{x+2} + 1$.

- The $y$-intercepts of the graphs are 14 units apart.
- Point A is a point of intersection as well as an $x$-intercept for both graphs.




2.1.1 Is $g(x)$ a function? Give a reason for your answer. (2K)

yes ✓

passes vertical line test or is a many-to-one mapping ✓

2.1.2 Is the inverse of $g(x)$ a function? Give a reason for your answer. (2K)

no ✓

doesn't pass vertical line test or is a one-to-many mapping ✓

2.1.3 Determine the coordinates of $A$. (3R)

$g(x) = x^2 + x - 12$

$0 = x^2 + x - 12$ ✓

$0 = (x + 4)(x - 3)$ ✓

$x = -4$ or $x = 3$ (N/A) ✓ $\therefore A(-4;0)$

2.1.4 Prove that $p = 2$. (3R)

If the $y$-intercepts of the graphs are 14 units apart, then
the $y$-intercept of $h(x)$ is $y = -12 + 14 = 2$ ✓

$$h(x) = \frac{p}{x+2} + 1$$

$$2 = \frac{p}{0+2} + 1 \checkmark$$

$$p = 2 \checkmark$$

2.1.5 Determine the axis of symmetry with a positive gradient of $h(x)$. (4C)

$m = 1$ (-2;1)

$y - y^1 = m(x - x_1)$ ✓

$y - 1 = 1(x + 2)$ ✓

$y = x + 2 + 1$

$y = x + 3$ ✓✓

2.2 Given: $f(x) = 3^x - 3$

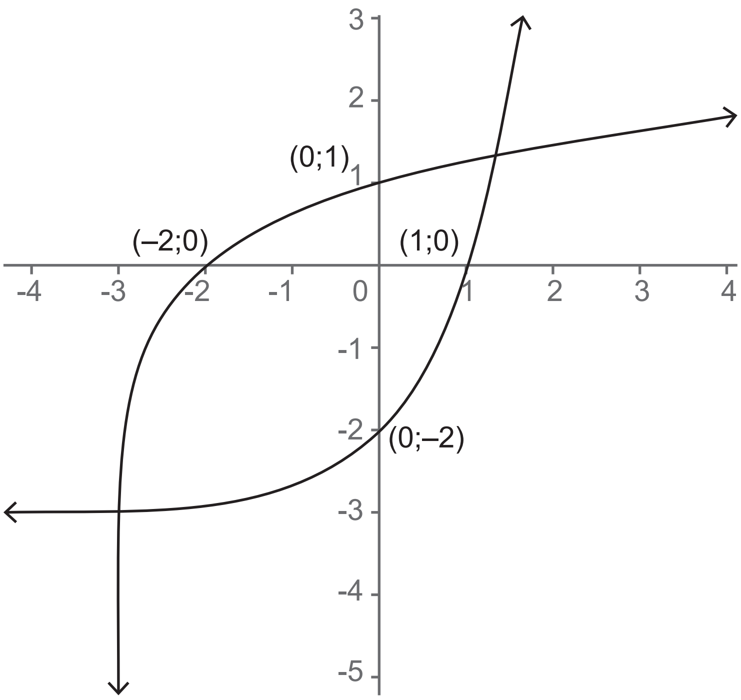
2.2.1 Write down the equation of $f^{-1}$, the inverse of $f$, in the form $y = ...$ (3R)

$x = 3^y - 3$ ✓

$3^y = x + 3$ ✓

$y = log_3 (x + 3)$✓

2.2.2 On the same set of axes, sketch the graphs of $f$ and $f^{-1}$, clearly showing all intercepts with the axes. (5R)




One mark for each correct intercept. (4 marks)

One mark for the correct shapes. (1 mark)

2.2.3 Determine the equation of the asymptote for $f^{-1}$. (1C)

$x = -3$ ✓

2.2.4 Write down the domain of both $f$ and $f^{-1}$. (2R)
Domain of $f$ : $x \in R$ ✓
Domain of $f^{-1}$ : $x > -3$ ✓

2.2.5 Determine the values of $k$ for which $f(x + 2) = k$ will have no solution. (2P)
$k < -3$ ✓ ✓ (by inspection)

# RESOURCE 14

## Term 1 Test 2

| QUESTION | DESCRIPTION | MAXIMUM MARK | ACTUAL MARK |
|---|---|---|---|
| 1 | Trigonometry | 32 | |
| 2 | Finance Growth and Decay | 18 | |
| | TOTAL | 50 | |

**QUESTION 1** **32 MARKS**

1.1 Simplify the following without the use of a calculator:

1.1.1 $1 - 2\,sin^2\,135°$ (2)

1.1.2 $8\,cos\,22{,}5°.\,cos\,67{,}5°$ (3)

1.2 Given: $5\,sin\,\alpha = 3$ and $tan\,\beta = -1$, where $\alpha, \beta \in (90°;270°)$.
Without using your calculator, determine the value of:

1.2.1 $cos\,\alpha$ (2)

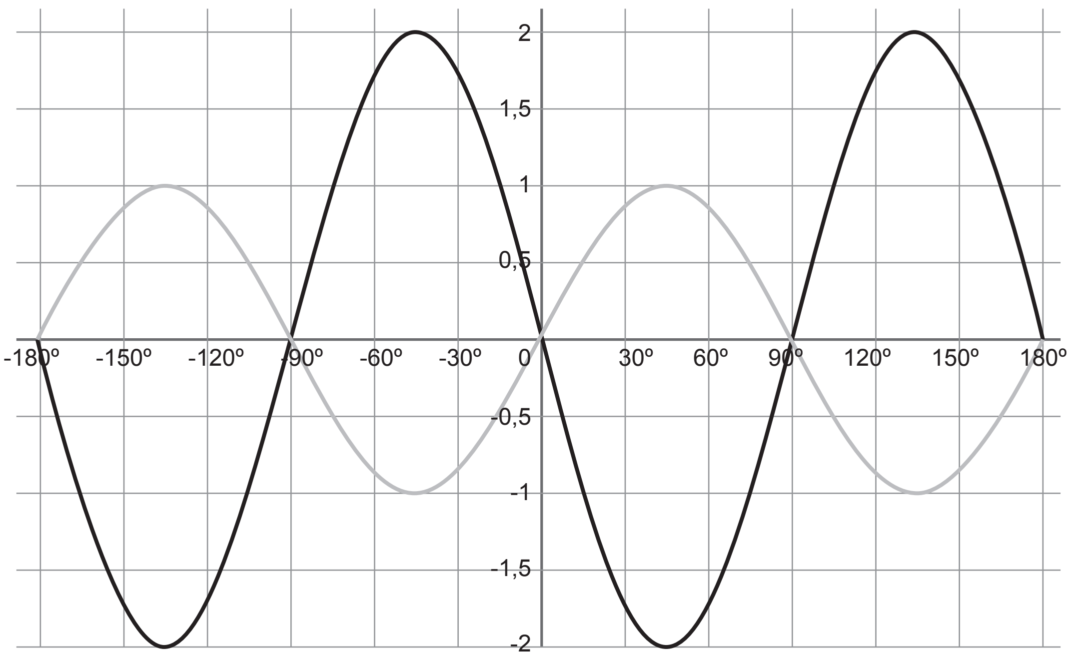
1.2.2 $sin\,2\beta$ (3)

1.2.3 $sin\,(\alpha + \beta)$ (5)

1.3 Determine the general solution of $sin\left(\frac{1}{2}x + 15°\right) = cos\,(2x - 15°)$ (5)

1.4 Prove that $\dfrac{2\,sin\,\theta.cos\,\theta + 1}{cos\,2\theta} = \dfrac{cos\,\theta + sin\,\theta}{cos\,\theta - sin\,\theta}$ (5)

1.5 The functions $f(x) = -2\,sin\,2x$ and $g(x) = cos\,(2x - 90°)$ are drawn for the interval $x \in [-180°; 180°]$




1.5.1 Write down the amplitude of $f(x)$. (1)

1.5.2 Write down the minimum value of $g(x) - 10$. (2)

1.5.3 Determine the values of x for which $f(x) \geq g(x)$. (4)

**QUESTION 2** **18 MARKS**

2.1 Ayanda wants to buy a new car that costs R225 000. He has not saved up enough money to pay a deposit and will need to take out a bank loan for the whole amount. The bank agrees to give Ayanda an interest rate of 9,6 % per annum compounded monthly over 6 years.

2.1.1 What will Ayanda's monthly instalments be? (4)

2.1.2 At the end of the 6 year period, how much interest would Ayanda have paid to the bank? (2)

2.1.3 What is the effective interest rate for the bank loan's interest. (3)

2.1.4 If Ayanda decides to pay off his loan by making monthly payment of R5 000, how long will it take him to finish paying off his loan (give your answer in months)? (5)

2.1.5 After considering all this, Ayanda decides to rather save his money until he has enough money to buy the car without having to take a loan. An investment company offers Ayanda a savings plan of 10,4% per annum compounded quarterly. How much should Ayanda's quarterly deposits into the savings plan be if he would like to buy the car in four years' time. (4)

# RESOURCE 11

## Memorandum Term 1 Test 2

| QUESTION | DESCRIPTION | MAXIMUM MARK | ACTUAL MARK |
|---|---|---|---|
| 1 | Trigonometry | 32 | |
| 2 | Finance Growth and Decay | 18 | |
| | TOTAL | 50 | |

**QUESTION 1** **32 MARKS**

1.1 Simplify the following without the use of a calculator:
Marking note: steps must be shown. Answer only must receive zero marks.

1.1.1 $1 - 2\sin^2 135^\circ$ (2K)

$= \cos 270^\circ$ ✓

$= 0$ ✓

or:

$= 1 - 2\sin^2 45^\circ$ ✓

$= 1 - 2\left(\frac{\sqrt{2}}{2}\right)^2$ ✓

$= 0$

1.1.2 $8\cos 22{,}5^\circ . \cos 67{,}5^\circ$ (3P)

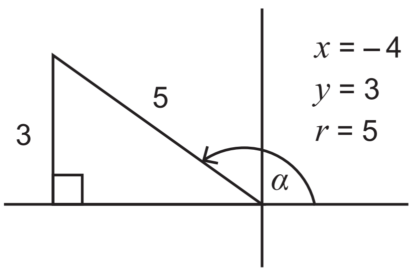
$= 8\cos 22{,}5^\circ . \sin 22{,}5^\circ$ ✓

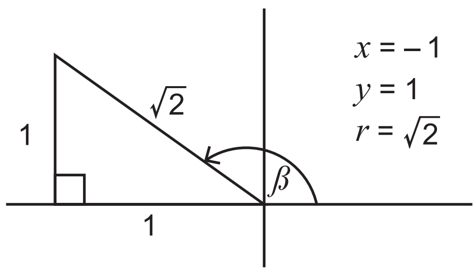
$= 4\sin 45^\circ$ ✓

$= 4\left(\frac{\sqrt{2}}{2}\right)$

$= 2\sqrt{2}$ ✓

1.2 Given: $5\sin\alpha = 3$ and $\tan\beta = -1$, where $\alpha, \beta \in (90^\circ; 270^\circ)$.
Without using your calculator, determine the value of:




1.2.1 $\cos\alpha$ (2K)

$= -\frac{4}{5}$ ✓ ✓

1.2.2 $\sin 2\beta$ (3K)

$= 2\sin\beta . \cos\beta$ ✓

$= 2\left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\right)$ ✓

$= -2$ ✓

1.2.3 $sin\,(\alpha + \beta)$ (5R)

$= sin\,\alpha.cos\,\beta + cos\,\alpha.sin\beta$ ✓

$= \left(\frac{3}{5}\right)\left(-\frac{1}{\sqrt{2}}\right)+\left(-\frac{4}{5}\right)\left(\frac{1}{\sqrt{2}}\right)$ ✓ ✓

$= -\frac{3}{5\sqrt{2}} - \frac{4}{5\sqrt{2}}$

$= -\frac{7}{5\sqrt{2}}$ ✓ $= -\frac{7\sqrt{2}}{10}$ ✓

1.3 Determine the general solution of $sin\,(\frac{1}{2}x + 15^\circ) = cos\,(2x - 15^\circ)$ (5C)

$sin\left(\frac{1}{2}x + 15^\circ\right) = cos\,(2x - 15^\circ)$

$sin\left(\frac{1}{2}x + 15^\circ\right) = sin\,[90^\circ - (2x - 15^\circ)]$ ✓

$sin\left(\frac{1}{2}x + 15^\circ\right) = sin\,(105^\circ - 2x)$

Quad 1

$\frac{1}{2}x + 15^\circ = 105^\circ - 2x + k.360^\circ$ ✓

$x + 30^\circ = 210^\circ - 4x + k.720^\circ$

$5x = 180^\circ + k.720^\circ$

$x = 36^\circ + k.144^\circ$ ✓

Quad 2

$\frac{1}{2}x + 15^\circ = 180^\circ - (105^\circ - 2x) + k.360^\circ$ ✓

$\frac{1}{2}x + 15^\circ = 75^\circ - 2x + k.360^\circ$

$x + 30^\circ = 150^\circ + 4x + k.720^\circ$

$-3x = 120^\circ + \text{k}.720^\circ$

$x = -40^\circ - k.240^\circ$ ✓

$k \in 2$

1.4 Prove that $\frac{2\,sin\,\theta.cos\,\theta + 1}{cos\,2\,\theta} = \frac{cos\,\theta + sin\,\theta}{cos\,\theta - sin\,\theta}$ (5C)

LHS $= \frac{2\,sin\,\theta.cos\,\theta + 1}{cos\,2\,\theta}$

$= \frac{2\,sin\,\theta.cos\,\theta + 1}{cos^2\theta - sin^2\theta}$ ✓

$= \frac{2\,sin\,\theta.cos\,\theta + sin^2\theta + cos^2\theta}{cos^2\theta - sin^2\theta}$ ✓

$= \frac{(sin\theta + cos\theta)(sin\,\theta + \cos\theta)}{\cos^2\theta - sin^2\theta}$ ✓

$= \frac{(sin\,\theta + cos\,\theta)(sin\,\theta + cos\,\theta)}{(cos\,\theta - sin\,\theta)(cos\,\theta + sin\,\theta)}$ ✓

$= \frac{(cos\,\theta + sin\,\theta)}{(cos\,\theta - sin\,\theta)}$ ✓

= RHS

1.5.1 Write down the amplitude of $f(x)$. (1K)

2 ✓

1.5.2 Write down the minimum value of $g(x) - 10$. (2K)

–11 ✓ ✓

1.5.3 Determine the values of $x$ for which $f(x) \geq g(x)$. (4K)

$x \in [-90°;0°]$ ✓ ✓ and $x \in [90°;180°]$ ✓ ✓

**QUESTION 2** **18 MARKS**

2.1 Ayanda wants to buy a new car that costs R225 000. He has not saved up enough money to pay a deposit and will need to take out a bank loan for the whole amount. The bank agrees to give Ayanda an interest rate of 9,6 % per annum compounded monthly over 6 years.

2.1.1 What will Ayanda's monthly instalments be? (4R)

$$P = \frac{x[1-(1+i)^{-n}]}{i} \checkmark$$

$$i = 0{,}096 \div 12 = 0{,}008 \checkmark$$

$$n = 12 \times 6 = 72 \checkmark$$

$$225000 = \frac{x[1-(1+0{,}008)^{-72}]}{0{,}008}$$

$$1800 = x[1-(1+0{,}008)^{-72}]$$

$$x = \text{R } 4123{,}07 \checkmark$$

2.1.2 At the end of the 6 year period, how much interest would Ayanda have paid to the bank? (2C)

4123,07 × 72 = 296 861,04 ✓

296 861,04 – 225 000 = R 71 861,04 ✓

2.1.3 What is the effective interest rate for the bank loan's interest. (3R)

$$i_{eff} + 1 = \left(1 + \frac{i_{nom}}{n}\right)^n \checkmark$$

$$i_{eff} = \left(1 + \frac{0{,}096}{12}\right)^{12} - 1 \checkmark$$

$$i_{eff} = 10{,}03\% \checkmark$$

2.1.4 If Ayanda decides to pay off his loan by making monthly payment of R5 000, how long will it take him to finish paying off his loan (give your answer in months)? (5C)

$$225000 = \frac{5000[1-(1+0,008)^{-n}]}{0,008} \checkmark$$

$$1800 = 5000[1-(1+0,008)^{-n}]$$

$$0,36 = 1-(1,008)^{-n}$$

$$-0,64 = -(1,008)^{-n}$$

$$0,64 = (1,008)^{-n} \checkmark$$

$$-n = log_{1,008} 0,64 \checkmark$$

$$-n = -56,01 \checkmark$$

$$n = 57 \text{ months} \checkmark$$

2.1.5 After considering all this, Ayanda decides to rather save his money until he has enough money to buy the car without having to take a loan.
An investment company offers Ayanda a savings plan of 10,4% per annum compounded quarterly. How much should Ayanda's quarterly deposits into the savings plan be if he would like to buy the car in four years' time. (4R)

$$i = 0,104 \div 4 = 0,026$$

$$n = 4 \times 4 = 16$$

$$F = \frac{x[(1+i)^n-1]}{i}$$

$$225000 = \frac{x[(1+0,026)^{16}-1]}{0,026} \checkmark\checkmark$$

$$5850 = x[(1,026)^{16}-1] \checkmark$$

$$x = \text{R}11\ 519,18 \checkmark$$